

10.2 Independent and Dependent Events

Essential Question How can you determine whether two events are independent or dependent?

Two events are **independent events** when the occurrence of one event does not affect the occurrence of the other event. Two events are **dependent events** when the occurrence of one event *does* affect the occurrence of the other event.

EXPLORATION 1 Identifying Independent and Dependent Events

Work with a partner. Determine whether the events are independent or dependent. Explain your reasoning.

- Two six-sided dice are rolled.
- Six pieces of paper, numbered 1 through 6, are in a bag. Two pieces of paper are selected one at a time without replacement.



REASONING ABSTRACTLY

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

EXPLORATION 2 Finding Experimental Probabilities

Work with a partner.

- In Exploration 1(a), experimentally estimate the probability that the sum of the two numbers rolled is 7. Describe your experiment.
- In Exploration 1(b), experimentally estimate the probability that the sum of the two numbers selected is 7. Describe your experiment.

EXPLORATION 3 Finding Theoretical Probabilities

Work with a partner.

- In Exploration 1(a), find the theoretical probability that the sum of the two numbers rolled is 7. Then compare your answer with the experimental probability you found in Exploration 2(a).
- In Exploration 1(b), find the theoretical probability that the sum of the two numbers selected is 7. Then compare your answer with the experimental probability you found in Exploration 2(b).
- Compare the probabilities you obtained in parts (a) and (b).

Communicate Your Answer

- How can you determine whether two events are independent or dependent?
- Determine whether the events are independent or dependent. Explain your reasoning.
 - You roll a 4 on a six-sided die and spin red on a spinner.
 - Your teacher chooses a student to lead a group, chooses another student to lead a second group, and chooses a third student to lead a third group.

10.2 Lesson

Core Vocabulary

independent events, p. 546
dependent events, p. 547
conditional probability, p. 547

Previous

probability
sample space

What You Will Learn

- ▶ Determine whether events are independent events.
- ▶ Find probabilities of independent and dependent events.
- ▶ Find conditional probabilities.

Determining Whether Events Are Independent

Two events are **independent events** when the occurrence of one event does not affect the occurrence of the other event.

Core Concept

Probability of Independent Events

Words Two events A and B are independent events if and only if the probability that both events occur is the product of the probabilities of the events.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

EXAMPLE 1 Determining Whether Events Are Independent

A student taking a quiz randomly guesses the answers to four true-false questions. Use a sample space to determine whether guessing Question 1 correctly and guessing Question 2 correctly are independent events.

SOLUTION

Using the sample space in Example 2 on page 539:

$$P(\text{correct on Question 1}) = \frac{8}{16} = \frac{1}{2} \quad P(\text{correct on Question 2}) = \frac{8}{16} = \frac{1}{2}$$

$$P(\text{correct on Question 1 and correct on Question 2}) = \frac{4}{16} = \frac{1}{4}$$

- ▶ Because $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, the events are independent.

EXAMPLE 2 Determining Whether Events Are Independent

A group of four students includes one boy and three girls. The teacher randomly selects one of the students to be the speaker and a different student to be the recorder. Use a sample space to determine whether randomly selecting a girl first and randomly selecting a girl second are independent events.

SOLUTION

Let B represent the boy. Let G_1 , G_2 , and G_3 represent the three girls. Use a table to list the outcomes in the sample space.

Using the sample space:

$$P(\text{girl first}) = \frac{9}{12} = \frac{3}{4} \quad P(\text{girl second}) = \frac{9}{12} = \frac{3}{4}$$

$$P(\text{girl first and girl second}) = \frac{6}{12} = \frac{1}{2}$$

- ▶ Because $\frac{3}{4} \cdot \frac{3}{4} \neq \frac{1}{2}$, the events are not independent.

| Number of girls | Outcome | |
|-----------------|----------|----------|
| 1 | G_1B | BG_1 |
| 1 | G_2B | BG_2 |
| 1 | G_3B | BG_3 |
| 2 | G_1G_2 | G_2G_1 |
| 2 | G_1G_3 | G_3G_1 |
| 2 | G_2G_3 | G_3G_2 |

1. In Example 1, determine whether guessing Question 1 incorrectly and guessing Question 2 correctly are independent events.
2. In Example 2, determine whether randomly selecting a girl first and randomly selecting a boy second are independent events.

Finding Probabilities of Events

In Example 1, it makes sense that the events are independent because the second guess should not be affected by the first guess. In Example 2, however, the selection of the second person *depends* on the selection of the first person because the same person cannot be selected twice. These events are *dependent*. Two events are **dependent events** when the occurrence of one event *does* affect the occurrence of the other event.

The probability that event B occurs given that event A has occurred is called the **conditional probability** of B given A and is written as $P(B|A)$.

MAKING SENSE OF PROBLEMS

One way that you can find $P(\text{girl second} | \text{girl first})$ is to list the 9 outcomes in which a girl is chosen first and then find the fraction of these outcomes in which a girl is chosen second:

| | | |
|----------|----------|----------|
| G_1B | G_2B | G_3B |
| G_1G_2 | G_2G_1 | G_3G_1 |
| G_1G_3 | G_2G_3 | G_3G_2 |

Core Concept

Probability of Dependent Events

Words If two events A and B are dependent events, then the probability that both events occur is the product of the probability of the first event and the conditional probability of the second event given the first event.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Example Using the information in Example 2:

$$\begin{aligned}
 P(\text{girl first and girl second}) &= P(\text{girl first}) \cdot P(\text{girl second} | \text{girl first}) \\
 &= \frac{9}{12} \cdot \frac{6}{9} = \frac{1}{2}
 \end{aligned}$$

EXAMPLE 3 Finding the Probability of Independent Events

As part of a board game, you need to spin the spinner, which is divided into equal parts. Find the probability that you get a 5 on your first spin and a number greater than 3 on your second spin.



SOLUTION

Let event A be “5 on first spin” and let event B be “greater than 3 on second spin.”

The events are independent because the outcome of your second spin is not affected by the outcome of your first spin. Find the probability of each event and then multiply the probabilities.

$$P(A) = \frac{1}{8} \quad \text{1 of the 8 sections is a "5."}$$

$$P(B) = \frac{5}{8} \quad \text{5 of the 8 sections (4, 5, 6, 7, 8) are greater than 3.}$$

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{1}{8} \cdot \frac{5}{8} = \frac{5}{64} \approx 0.078$$

- So, the probability that you get a 5 on your first spin and a number greater than 3 on your second spin is about 7.8%.



EXAMPLE 4 Finding the Probability of Dependent Events

A bag contains twenty \$1 bills and five \$100 bills. You randomly draw a bill from the bag, set it aside, and then randomly draw another bill from the bag. Find the probability that both events A and B will occur.

Event A : The first bill is \$100. **Event B :** The second bill is \$100.

SOLUTION

The events are dependent because there is one less bill in the bag on your second draw than on your first draw. Find $P(A)$ and $P(B|A)$. Then multiply the probabilities.

$$P(A) = \frac{5}{25} \quad \text{5 of the 25 bills are \$100 bills.}$$

$$P(B|A) = \frac{4}{24} \quad \text{4 of the remaining 24 bills are \$100 bills.}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{5}{25} \cdot \frac{4}{24} = \frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30} \approx 0.033.$$

► So, the probability that you draw two \$100 bills is about 3.3%.

EXAMPLE 5 Comparing Independent and Dependent Events

You randomly select 3 cards from a standard deck of 52 playing cards. What is the probability that all 3 cards are hearts when (a) you replace each card before selecting the next card, and (b) you do not replace each card before selecting the next card? Compare the probabilities.

SOLUTION

Let event A be “first card is a heart,” event B be “second card is a heart,” and event C be “third card is a heart.”

a. Because you replace each card before you select the next card, the events are independent. So, the probability is

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C) = \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{64} \approx 0.016.$$

b. Because you do not replace each card before you select the next card, the events are dependent. So, the probability is

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B|A) \cdot P(C|A \text{ and } B)$$

$$= \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{11}{850} \approx 0.013.$$

► So, you are $\frac{1}{64} \div \frac{11}{850} \approx 1.2$ times more likely to select 3 hearts when you replace each card before you select the next card.

STUDY TIP

The formulas for finding probabilities of independent and dependent events can be extended to three or more events.



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- In Example 3, what is the probability that you spin an even number and then an odd number?
- In Example 4, what is the probability that both bills are \$1 bills?
- In Example 5, what is the probability that none of the cards drawn are hearts when (a) you replace each card, and (b) you do not replace each card? Compare the probabilities.

Finding Conditional Probabilities

EXAMPLE 6 Using a Table to Find Conditional Probabilities

| | Pass | Fail |
|---------------|------|------|
| Defective | 3 | 36 |
| Non-defective | 450 | 11 |

A quality-control inspector checks for defective parts. The table shows the results of the inspector's work. Find (a) the probability that a defective part "passes," and (b) the probability that a non-defective part "fails."

SOLUTION

$$\begin{aligned} \text{a. } P(\text{pass} | \text{defective}) &= \frac{\text{Number of defective parts "passed"}}{\text{Total number of defective parts}} \\ &= \frac{3}{3 + 36} = \frac{3}{39} = \frac{1}{13} \approx 0.077, \text{ or about } 7.7\% \end{aligned}$$

$$\begin{aligned} \text{b. } P(\text{fail} | \text{non-defective}) &= \frac{\text{Number of non-defective parts "failed"}}{\text{Total number of non-defective parts}} \\ &= \frac{11}{450 + 11} = \frac{11}{461} \approx 0.024, \text{ or about } 2.4\% \end{aligned}$$

STUDY TIP

Note that when A and B are independent, this rule still applies because $P(B) = P(B|A)$.

You can rewrite the formula for the probability of dependent events to write a rule for finding conditional probabilities.

$$P(A) \cdot P(B|A) = P(A \text{ and } B) \quad \text{Write formula.}$$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{Divide each side by } P(A).$$

EXAMPLE 7 Finding a Conditional Probability

At a school, 60% of students buy a school lunch. Only 10% of students buy lunch and dessert. What is the probability that a student who buys lunch also buys dessert?

SOLUTION

Let event A be "buys lunch" and let event B be "buys dessert." You are given $P(A) = 0.6$ and $P(A \text{ and } B) = 0.1$. Use the formula to find $P(B|A)$.

$$\begin{aligned} P(B|A) &= \frac{P(A \text{ and } B)}{P(A)} && \text{Write formula for conditional probability.} \\ &= \frac{0.1}{0.6} && \text{Substitute 0.1 for } P(A \text{ and } B) \text{ and 0.6 for } P(A). \\ &= \frac{1}{6} \approx 0.167 && \text{Simplify.} \end{aligned}$$

► So, the probability that a student who buys lunch also buys dessert is about 16.7%.

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- In Example 6, find (a) the probability that a non-defective part "passes," and (b) the probability that a defective part "fails."
- At a coffee shop, 80% of customers order coffee. Only 15% of customers order coffee and a bagel. What is the probability that a customer who orders coffee also orders a bagel?

Vocabulary and Core Concept Check

- WRITING** Explain the difference between dependent events and independent events, and give an example of each.
- COMPLETE THE SENTENCE** The probability that event B will occur given that event A has occurred is called the _____ of B given A and is written as _____.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, tell whether the events are independent or dependent. Explain your reasoning.

- A box of granola bars contains an assortment of flavors. You randomly choose a granola bar and eat it. Then you randomly choose another bar.

Event A: You choose a coconut almond bar first.

Event B: You choose a cranberry almond bar second.

- You roll a six-sided die and flip a coin.

Event A: You get a 4 when rolling the die.

Event B: You get tails when flipping the coin.



- Your MP3 player contains hip-hop and rock songs. You randomly choose a song. Then you randomly choose another song without repeating song choices.

Event A: You choose a hip-hop song first.

Event B: You choose a rock song second.



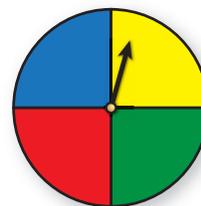
- There are 22 novels of various genres on a shelf. You randomly choose a novel and put it back. Then you randomly choose another novel.

Event A: You choose a mystery novel.

Event B: You choose a science fiction novel.

In Exercises 7–10, determine whether the events are independent. (See Examples 1 and 2.)

- You play a game that involves spinning a wheel. Each section of the wheel shown has the same area. Use a sample space to determine whether randomly spinning blue and then green are independent events.

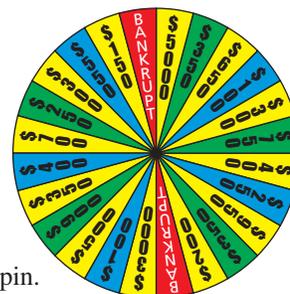


- You have one red apple and three green apples in a bowl. You randomly select one apple to eat now and another apple for your lunch. Use a sample space to determine whether randomly selecting a green apple first and randomly selecting a green apple second are independent events.

- A student is taking a multiple-choice test where each question has four choices. The student randomly guesses the answers to the five-question test. Use a sample space to determine whether guessing Question 1 correctly and Question 2 correctly are independent events.

- A vase contains four white roses and one red rose. You randomly select two roses to take home. Use a sample space to determine whether randomly selecting a white rose first and randomly selecting a white rose second are independent events.

- PROBLEM SOLVING** You play a game that involves spinning the money wheel shown. You spin the wheel twice. Find the probability that you get more than \$500 on your first spin and then go bankrupt on your second spin. (See Example 3.)



12. **PROBLEM SOLVING** You play a game that involves drawing two numbers from a hat. There are 25 pieces of paper numbered from 1 to 25 in the hat. Each number is replaced after it is drawn. Find the probability that you will draw the 3 on your first draw and a number greater than 10 on your second draw.

13. **PROBLEM SOLVING** A drawer contains 12 white socks and 8 black socks. You randomly choose 1 sock and do not replace it. Then you randomly choose another sock. Find the probability that both events A and B will occur. (See Example 4.)

Event A: The first sock is white.

Event B: The second sock is white.

14. **PROBLEM SOLVING** A word game has 100 tiles, 98 of which are letters and 2 of which are blank. The numbers of tiles of each letter are shown. You randomly draw 1 tile, set it aside, and then randomly draw another tile. Find the probability that both events A and B will occur.

Event A:
The first tile is a consonant.

Event B:
The second tile is a vowel.

| | | | | | | | |
|---|----|---|---|---|---|-------|---|
| A | 9 | H | 2 | O | 8 | V | 2 |
| B | 2 | I | 9 | P | 2 | W | 2 |
| C | 2 | J | 1 | Q | 1 | X | 1 |
| D | 4 | K | 1 | R | 6 | Y | 2 |
| E | 12 | L | 4 | S | 4 | Z | 1 |
| F | 2 | M | 2 | T | 6 | | 2 |
| G | 3 | N | 6 | U | 4 | Blank | |

15. **ERROR ANALYSIS** Events A and B are independent. Describe and correct the error in finding $P(A \text{ and } B)$.



$$P(A) = 0.6 \quad P(B) = 0.2$$

$$P(A \text{ and } B) = 0.6 + 0.2 = 0.8$$

16. **ERROR ANALYSIS** A shelf contains 3 fashion magazines and 4 health magazines. You randomly choose one to read, set it aside, and randomly choose another for your friend to read. Describe and correct the error in finding the probability that both events A and B occur.

Event A: The first magazine is fashion.

Event B: The second magazine is health.



$$P(A) = \frac{3}{7} \quad P(B|A) = \frac{4}{7}$$

$$P(A \text{ and } B) = \frac{3}{7} \cdot \frac{4}{7} = \frac{12}{49} \approx 0.245$$

17. **NUMBER SENSE** Events A and B are independent. Suppose $P(B) = 0.4$ and $P(A \text{ and } B) = 0.13$. Find $P(A)$.

18. **NUMBER SENSE** Events A and B are dependent. Suppose $P(B|A) = 0.6$ and $P(A \text{ and } B) = 0.15$. Find $P(A)$.

19. **ANALYZING RELATIONSHIPS** You randomly select three cards from a standard deck of 52 playing cards. What is the probability that all three cards are face cards when (a) you replace each card before selecting the next card, and (b) you do not replace each card before selecting the next card? Compare the probabilities. (See Example 5.)

20. **ANALYZING RELATIONSHIPS** A bag contains 9 red marbles, 4 blue marbles, and 7 yellow marbles. You randomly select three marbles from the bag. What is the probability that all three marbles are red when (a) you replace each marble before selecting the next marble, and (b) you do not replace each marble before selecting the next marble? Compare the probabilities.

21. **ATTEND TO PRECISION** The table shows the number of species in the United States listed as endangered and threatened. Find (a) the probability that a randomly selected endangered species is a bird, and (b) the probability that a randomly selected mammal is endangered. (See Example 6.)

| | Endangered | Threatened |
|---------|------------|------------|
| Mammals | 70 | 16 |
| Birds | 80 | 16 |
| Other | 318 | 142 |

22. **ATTEND TO PRECISION** The table shows the number of tropical cyclones that formed during the hurricane seasons over a 12-year period. Find (a) the probability to predict whether a future tropical cyclone in the Northern Hemisphere is a hurricane, and (b) the probability to predict whether a hurricane is in the Southern Hemisphere.

| Type of Tropical Cyclone | Northern Hemisphere | Southern Hemisphere |
|--------------------------|---------------------|---------------------|
| tropical depression | 100 | 107 |
| tropical storm | 342 | 487 |
| hurricane | 379 | 525 |

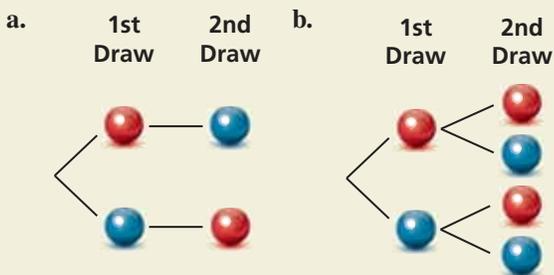
23. **PROBLEM SOLVING** At a school, 43% of students attend the homecoming football game. Only 23% of students go to the game and the homecoming dance. What is the probability that a student who attends the football game also attends the dance? (See Example 7.)

24. **PROBLEM SOLVING** At a gas station, 84% of customers buy gasoline. Only 5% of customers buy gasoline and a beverage. What is the probability that a customer who buys gasoline also buys a beverage?

25. **PROBLEM SOLVING** You and 19 other students volunteer to present the “Best Teacher” award at a school banquet. One student volunteer will be chosen to present the award. Each student worked at least 1 hour in preparation for the banquet. You worked for 4 hours, and the group worked a combined total of 45 hours. For each situation, describe a process that gives you a “fair” chance to be chosen, and find the probability that you are chosen.

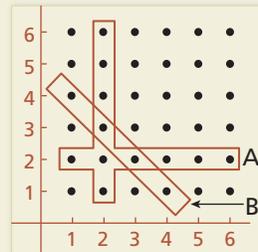
- “Fair” means equally likely.
- “Fair” means proportional to the number of hours each student worked in preparation.

26. **HOW DO YOU SEE IT?** A bag contains one red marble and one blue marble. The diagrams show the possible outcomes of randomly choosing two marbles using different methods. For each method, determine whether the marbles were selected with or without replacement.



27. **MAKING AN ARGUMENT** A meteorologist claims that there is a 70% chance of rain. When it rains, there is a 75% chance that your softball game will be rescheduled. Your friend believes the game is more likely to be rescheduled than played. Is your friend correct? Explain your reasoning.

28. **THOUGHT PROVOKING** Two six-sided dice are rolled once. Events A and B are represented by the diagram. Describe each event. Are the two events dependent or independent? Justify your reasoning.



29. **MODELING WITH MATHEMATICS** A football team is losing by 14 points near the end of a game. The team scores two touchdowns (worth 6 points each) before the end of the game. After each touchdown, the coach must decide whether to go for 1 point with a kick (which is successful 99% of the time) or 2 points with a run or pass (which is successful 45% of the time).



- If the team goes for 1 point after each touchdown, what is the probability that the team wins? loses? ties?
- If the team goes for 2 points after each touchdown, what is the probability that the team wins? loses? ties?
- Can you develop a strategy so that the coach’s team has a probability of winning the game that is greater than the probability of losing? If so, explain your strategy and calculate the probabilities of winning and losing the game.

30. **ABSTRACT REASONING** Assume that A and B are independent events.

- Explain why $P(B) = P(B|A)$ and $P(A) = P(A|B)$.
- Can $P(A \text{ and } B)$ also be defined as $P(B) \cdot P(A|B)$? Justify your reasoning.

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. (*Skills Review Handbook*)

31. $\frac{9}{10}x = 0.18$

32. $\frac{1}{4}x + 0.5x = 1.5$

33. $0.3x - \frac{3}{5}x + 1.6 = 1.555$