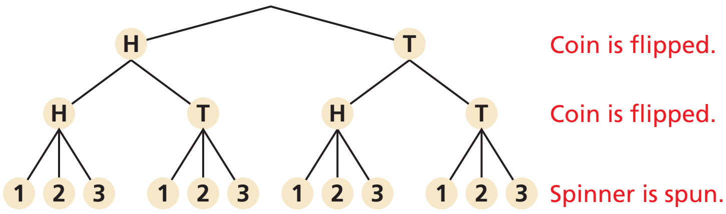
10.5 Permutations and Combinations

Essential Question

How can a tree diagram help you visualize the number of ways in which two or more events can occur?

EXPLORATION 1 Reading a Tree Diagram

Work with a partner. Two coins are flipped and the spinner is spun. The tree diagram shows the possible outcomes.

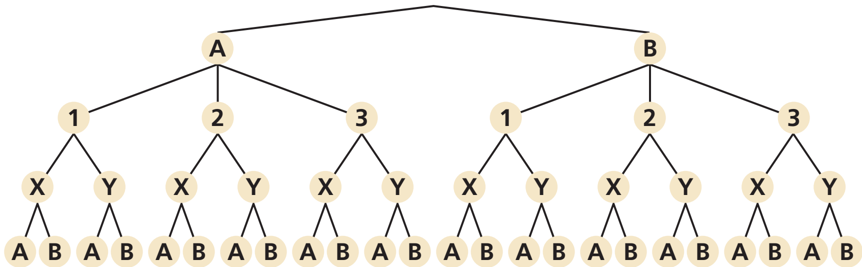


a. How many outcomes are possible?

b. List the possible outcomes.

EXPLORATION 2 Reading a Tree Diagram

Work with a partner. Consider the tree diagram below.



a. How many events are shown?

b. What outcomes are possible for each event?

c. How many outcomes are possible?

d. List the possible outcomes.

EXPLORATION 3 Writing a Conjecture

Work with a partner.

a. Consider the following general problem: Event 1 can occur in m ways and event 2 can occur in n ways. Write a conjecture about the number of ways the two events can occur. Explain your reasoning.

b. Use the conjecture you wrote in part (a) to write a conjecture about the number of ways *more than* two events can occur. Explain your reasoning.

c. Use the results of Explorations 1(a) and 2(c) to verify your conjectures.

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.

Communicate Your Answer

4. How can a tree diagram help you visualize the number of ways in which two or more events can occur?

5. In Exploration 1, the spinner is spun a second time. How many outcomes are possible?

10.5 Lesson

Core Vocabulary

permutation, *p. 570*
n factorial, *p. 570*
combination, *p. 572*
Binomial Theorem, *p. 574*

Previous
Fundamental Counting Principle
Pascal's Triangle

What You Will Learn

- Use the formula for the number of permutations.
- Use the formula for the number of combinations.
- Use combinations and the Binomial Theorem to expand binomials.

Permutations

A **permutation** is an arrangement of objects in which order is important. For instance, the 6 possible permutations of the letters A, B, and C are shown.

ABC ACB BAC BCA CAB CBA

EXAMPLE 1 Counting Permutations

Consider the number of permutations of the letters in the word JULY. In how many ways can you arrange (a) all of the letters and (b) 2 of the letters?

SOLUTION

a. Use the Fundamental Counting Principle to find the number of permutations of the letters in the word JULY.

$$\begin{aligned}\text{Number of permutations} &= \left(\begin{matrix}\text{Choices for}\\ \text{1st letter}\end{matrix}\right)\left(\begin{matrix}\text{Choices for}\\ \text{2nd letter}\end{matrix}\right)\left(\begin{matrix}\text{Choices for}\\ \text{3rd letter}\end{matrix}\right)\left(\begin{matrix}\text{Choices for}\\ \text{4th letter}\end{matrix}\right)\\ &= 4 \cdot 3 \cdot 2 \cdot 1\\ &= 24\end{aligned}$$

- There are 24 ways you can arrange all of the letters in the word JULY.

b. When arranging 2 letters of the word JULY, you have 4 choices for the first letter and 3 choices for the second letter.

$$\begin{aligned}\text{Number of permutations} &= \left(\begin{matrix}\text{Choices for}\\ \text{1st letter}\end{matrix}\right)\left(\begin{matrix}\text{Choices for}\\ \text{2nd letter}\end{matrix}\right)\\ &= 4 \cdot 3\\ &= 12\end{aligned}$$

- There are 12 ways you can arrange 2 of the letters in the word JULY.

REMEMBER

Fundamental Counting Principle: If one event can occur in *m* ways and another event can occur in *n* ways, then the number of ways that both events can occur is $m \cdot n$. The Fundamental Counting Principle can be extended to three or more events.

Monitoring Progress

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1. In how many ways can you arrange the letters in the word HOUSE?

2. In how many ways can you arrange 3 of the letters in the word MARCH?

In Example 1(a), you evaluated the expression $4 \cdot 3 \cdot 2 \cdot 1$. This expression can be written as 4! and is read "4 *factorial*." For any positive integer *n*, the product of the integers from 1 to *n* is called ***n* factorial** and is written as

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1.$$

As a special case, the value of 0! is defined to be 1.

In Example 1(b), you found the permutations of 4 objects taken 2 at a time. You can find the number of permutations using the formulas on the next page.

Core Concept

Permutations

Formulas	Examples
The number of permutations of n objects is given by $${}_nP_n = n!.$$	The number of permutations of 4 objects is $${}_4P_4 = 4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24.$$
The number of permutations of n objects taken r at a time, where $r \le n$, is given by $${}_nP_r = \frac{n!}{(n-r)!}.$$	The number of permutations of 4 objects taken 2 at a time is $${}_4P_2 = \frac{4!}{(4-2)!} = \frac{4 \bullet 3 \bullet \cancel{2!}}{\cancel{2!}} = 12.$$

USING A GRAPHING CALCULATOR

Most graphing calculators can calculate permutations.

```
4  nPr  4
             24
4  nPr  2
             12
```

EXAMPLE 2 Using a Permutations Formula

Ten horses are running in a race. In how many different ways can the horses finish first, second, and third? (Assume there are no ties.)

SOLUTION

To find the number of permutations of 3 horses chosen from 10, find ${}_{10}P_3$.

$$\begin{aligned}
{}_{10}P_3 &= \frac{10!}{(10-3)!} && \text{Permutations formula}\\
&= \frac{10!}{7!} && \text{Subtract.}\\
&= \frac{10 \bullet 9 \bullet 8 \bullet \cancel{7!}}{\cancel{7!}} && \text{Expand factorial. Divide out common factor, 7!.}\\
&= 720 && \text{Simplify.}
\end{aligned}$$

▶ There are 720 ways for the horses to finish first, second, and third.

STUDY TIP

When you divide out common factors, remember that 7! is a factor of 10!.

EXAMPLE 3 Finding a Probability Using Permutations

For a town parade, you will ride on a float with your soccer team. There are 12 floats in the parade, and their order is chosen at random. Find the probability that your float is first and the float with the school chorus is second.

SOLUTION

Step 1 Write the number of possible outcomes as the number of permutations of the 12 floats in the parade. This is ${}_{12}P_{12} = 12!$.

Step 2 Write the number of favorable outcomes as the number of permutations of the other floats, given that the soccer team is first and the chorus is second. This is ${}_{10}P_{10} = 10!$.

Step 3 Find the probability.

$$\begin{aligned}
P(\text{soccer team is 1st, chorus is 2nd}) &= \frac{10!}{12!} && \text{Form a ratio of favorable to possible outcomes.}\\
&= \frac{\cancel{10!}}{12 \bullet 11 \bullet \cancel{10!}} && \text{Expand factorial. Divide out common factor, 10!.}\\
&= \frac{1}{132} && \text{Simplify.}
\end{aligned}$$

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3. **WHAT IF?** In Example 2, suppose there are 8 horses in the race. In how many different ways can the horses finish first, second, and third? (Assume there are no ties.)

4. **WHAT IF?** In Example 3, suppose there are 14 floats in the parade. Find the probability that the soccer team is first and the chorus is second.

Combinations

A **combination** is a selection of objects in which order is *not* important. For instance, in a drawing for 3 identical prizes, you would use combinations, because the order of the winners would not matter. If the prizes were different, then you would use permutations, because the order would matter.

EXAMPLE 4 Counting Combinations

Count the possible combinations of 2 letters chosen from the list A, B, C, D.

SOLUTION

List all of the permutations of 2 letters from the list A, B, C, D. Because order is not important in a combination, cross out any duplicate pairs.

AB	AC	AD	~~BA~~	BC	BD
~~CA~~	~~CB~~	CD	~~DA~~	~~DB~~	~~DC~~

BD and DB are the same pair.

▶ There are 6 possible combinations of 2 letters from the list A, B, C, D.

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5. Count the possible combinations of 3 letters chosen from the list A, B, C, D, E.

In Example 4, you found the number of combinations of objects by making an organized list. You can also find the number of combinations using the following formula.

USING A GRAPHING CALCULATOR

Most graphing calculators can calculate combinations.

```
4  nCr  2
            6
```

Core Concept

Combinations

Formula The number of combinations of n objects taken r at a time, where $r \leq n$, is given by

$$_{n}C_{r} = \frac{n!}{(n - r)! \cdot r!}.$$

Example The number of combinations of 4 objects taken 2 at a time is

$$_{4}C_{2} = \frac{4!}{(4 - 2)! \cdot 2!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!} \cdot (2 \cdot 1)} = 6.$$

EXAMPLE 5 Using the Combinations Formula

You order a sandwich at a restaurant. You can choose 2 side dishes from a list of 8. How many combinations of side dishes are possible?

SOLUTION

The order in which you choose the side dishes is not important. So, to find the number of combinations of 8 side dishes taken 2 at a time, find ${}_8C_2$.

Check

```
8 nCr 2
             28
```

$$
\begin{aligned}
{}_8C_2 &= \frac{8!}{(8-2)! \cdot 2!} && \text{Combinations formula} \\
&= \frac{8!}{6! \cdot 2!} && \text{Subtract.} \\
&= \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot (2 \cdot 1)} && \text{Expand factorials. Divide out common factor, 6!.} \\
&= 28 && \text{Multiply.}
\end{aligned}
$$

▶ There are 28 different combinations of side dishes you can order.

EXAMPLE 6 Finding a Probability Using Combinations

A yearbook editor has selected 14 photos, including one of you and one of your friend, to use in a collage for the yearbook. The photos are placed at random. There is room for 2 photos at the top of the page. What is the probability that your photo and your friend's photo are the 2 placed at the top of the page?

SOLUTION

Step 1 Write the number of possible outcomes as the number of combinations of 14 photos taken 2 at a time, or ${}_{14}C_2$, because the order in which the photos are chosen is not important.

$$
\begin{aligned}
{}_{14}C_2 &= \frac{14!}{(14-2)! \cdot 2!} && \text{Combinations formula} \\
&= \frac{14!}{12! \cdot 2!} && \text{Subtract.} \\
&= \frac{14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot (2 \cdot 1)} && \text{Expand factorials. Divide out common factor, 12!.} \\
&= 91 && \text{Multiply.}
\end{aligned}
$$

Step 2 Find the number of favorable outcomes. Only one of the possible combinations includes your photo and your friend's photo.

Step 3 Find the probability.

$$P(\text{your photo and your friend's photos are chosen}) = \frac{1}{91}$$

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6. **WHAT IF?** In Example 5, suppose you can choose 3 side dishes out of the list of 8 side dishes. How many combinations are possible?

7. **WHAT IF?** In Example 6, suppose there are 20 photos in the collage. Find the probability that your photo and your friend's photo are the 2 placed at the top of the page.

Binomial Expansions

In Section 4.2, you used Pascal's Triangle to find binomial expansions. The table shows that the coefficients in the expansion of $(a + b)^n$ correspond to combinations.

	n	Pascal's Triangle as Numbers	Pascal's Triangle as Combinations	Binomial Expansion
0th row	0	1	${}_0C_0$	$(a + b)^0 = 1$
1st row	1	1 1	${}_1C_0 \quad {}_1C_1$	$(a + b)^1 = 1a + 1b$
2nd row	2	1 2 1	${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$	$(a + b)^2 = 1a^2 + 2ab + 1b^2$
3rd row	3	1 3 3 1	${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$	$(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

The results in the table are generalized in the **Binomial Theorem**.

Core Concept

The Binomial Theorem

For any positive integer n, the binomial expansion of $(a + b)^n$ is

$$(a + b)^n = {}_nC_0\, a^n b^0 + {}_nC_1\, a^{n-1} b^1 + {}_nC_2\, a^{n-2} b^2 + \cdots + {}_nC_n\, a^0 b^n.$$

Notice that each term in the expansion of $(a + b)^n$ has the form ${}_nC_r\, a^{n-r} b^r$, where r is an integer from 0 to n.

EXAMPLE 7 Using the Binomial Theorem

a. Use the Binomial Theorem to write the expansion of $(x^2 + y)^3$.

b. Find the coefficient of x^4 in the expansion of $(3x + 2)^{10}$.

SOLUTION

a.
$$\begin{aligned}(x^2 + y)^3 &= {}_3C_0(x^2)^3y^0 + {}_3C_1(x^2)^2y^1 + {}_3C_2(x^2)^1y^2 + {}_3C_3(x^2)^0y^3\\ &= (1)(x^6)(1) + (3)(x^4)(y^1) + (3)(x^2)(y^2) + (1)(1)(y^3)\\ &= x^6 + 3x^4y + 3x^2y^2 + y^3\end{aligned}$$

b. From the Binomial Theorem, you know

$$(3x + 2)^{10} = {}_{10}C_0(3x)^{10}(2)^0 + {}_{10}C_1(3x)^9(2)^1 + \cdots + {}_{10}C_{10}(3x)^0(2)^{10}.$$

Each term in the expansion has the form ${}_{10}C_r(3x)^{10-r}(2)^r$. The term containing x^4 occurs when $r = 6$.

$${}_{10}C_6(3x)^4(2)^6 = (210)(81x^4)(64) = 1{,}088{,}640x^4$$

► The coefficient of x^4 is 1,088,640.

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8. Use the Binomial Theorem to write the expansion of (a) $(x + 3)^5$ and (b) $(2p - q)^4$.

9. Find the coefficient of x^5 in the expansion of $(x - 3)^7$.

10. Find the coefficient of x^3 in the expansion of $(2x + 5)^8$.

10.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** An arrangement of objects in which order is important is called a(n) __________.

2. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$\dfrac{7!}{2! \cdot 5!}$	${}_7C_5$	${}_7C_2$	$\dfrac{7!}{(7-2)!}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the number of ways you can arrange (a) all of the letters and (b) 2 of the letters in the given word. *(See Example 1.)*

3. AT
4. TRY
5. ROCK
6. WATER
7. FAMILY
8. FLOWERS

In Exercises 9–16, evaluate the expression.

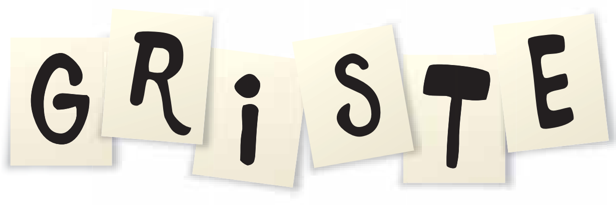
9. ${}_5P_2$
10. ${}_7P_3$
11. ${}_9P_1$
12. ${}_6P_5$
13. ${}_8P_6$
14. ${}_{12}P_0$
15. ${}_{30}P_2$
16. ${}_{25}P_5$

17. **PROBLEM SOLVING** Eleven students are competing in an art contest. In how many different ways can the students finish first, second, and third? *(See Example 2.)*

18. **PROBLEM SOLVING** Six friends go to a movie theater. In how many different ways can they sit together in a row of 6 empty seats?

19. **PROBLEM SOLVING** You and your friend are 2 of 8 servers working a shift in a restaurant. At the beginning of the shift, the manager randomly assigns one section to each server. Find the probability that you are assigned Section 1 and your friend is assigned Section 2. *(See Example 3.)*

20. **PROBLEM SOLVING** You make 6 posters to hold up at a basketball game. Each poster has a letter of the word TIGERS. You and 5 friends sit next to each other in a row. The posters are distributed at random. Find the probability that TIGERS is spelled correctly when you hold up the posters.



In Exercises 21–24, count the possible combinations of *r* letters chosen from the given list. *(See Example 4.)*

21. A, B, C, D; $r = 3$
22. L, M, N, O; $r = 2$
23. U, V, W, X, Y, Z; $r = 3$
24. D, E, F, G, H; $r = 4$

In Exercises 25–32, evaluate the expression.

25. ${}_5C_1$
26. ${}_8C_5$
27. ${}_9C_9$
28. ${}_8C_6$
29. ${}_{12}C_3$
30. ${}_{11}C_4$
31. ${}_{15}C_8$
32. ${}_{20}C_5$

33. **PROBLEM SOLVING** Each year, 64 golfers participate in a golf tournament. The golfers play in groups of 4. How many groups of 4 golfers are possible? *(See Example 5.)*

34. **PROBLEM SOLVING** You want to purchase vegetable dip for a party. A grocery store sells 7 different flavors of vegetable dip. You have enough money to purchase 2 flavors. How many combinations of 2 flavors of vegetable dip are possible?

ERROR ANALYSIS **In Exercises 35 and 36, describe and correct the error in evaluating the expression.**

35.

$$_{11}P_7 = \frac{11!}{(11-7)} = \frac{11!}{4} = 9{,}979{,}200$$

36.

$$_9C_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 3024$$

REASONING **In Exercises 37–40, tell whether the question can be answered using *permutations* or *combinations*. Explain your reasoning. Then answer the question.**

37. To complete an exam, you must answer 8 questions from a list of 10 questions. In how many ways can you complete the exam?

38. Ten students are auditioning for 3 different roles in a play. In how many ways can the 3 roles be filled?

39. Fifty-two athletes are competing in a bicycle race. In how many orders can the bicyclists finish first, second, and third? (Assume there are no ties.)

40. An employee at a pet store needs to catch 5 tetras in an aquarium containing 27 tetras. In how many groupings can the employee capture 5 tetras?

41. **CRITICAL THINKING** Compare the quantities $_{50}C_9$ and $_{50}C_{41}$ without performing any calculations. Explain your reasoning.

42. **CRITICAL THINKING** Show that each identity is true for any whole numbers r and n, where $0 \le r \le n$.

a. $_nC_n = 1$

b. $_nC_r = {}_nC_{n-r}$

c. $_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$

43. **REASONING** Consider a set of 4 objects.

a. Are there more permutations of all 4 of the objects or of 3 of the objects? Explain your reasoning.

b. Are there more combinations of all 4 of the objects or of 3 of the objects? Explain your reasoning.

c. Compare your answers to parts (a) and (b).

44. **OPEN-ENDED** Describe a real-life situation where the number of possibilities is given by $_5P_2$. Then describe a real-life situation that can be modeled by $_5C_2$.

45. **REASONING** Complete the table for each given value of r. Then write an inequality relating $_nP_r$ and $_nC_r$. Explain your reasoning.

	$r = 0$	$r = 1$	$r = 2$	$r = 3$
$_3P_r$				
$_3C_r$				

46. **REASONING** Write an equation that relates $_nP_r$ and $_nC_r$. Then use your equation to find and interpret the value of $\dfrac{_{182}P_4}{_{182}C_4}$.

47. **PROBLEM SOLVING** You and your friend are in the studio audience on a television game show. From an audience of 300 people, 2 people are randomly selected as contestants. What is the probability that you and your friend are chosen? *(See Example 6.)*

48. **PROBLEM SOLVING** You work 5 evenings each week at a bookstore. Your supervisor assigns you 5 evenings at random from the 7 possibilities. What is the probability that your schedule does not include working on the weekend?

REASONING In Exercises 49 and 50, find the probability of winning a lottery using the given rules. Assume that lottery numbers are selected at random.

49. You must correctly select 6 numbers, each an integer from 0 to 49. The order is not important.

50. You must correctly select 4 numbers, each an integer from 0 to 9. The order is important.

In Exercises 51–58, use the Binomial Theorem to write the binomial expansion. *(See Example 7a.)*

51. $(x + 2)^3$

52. $(c - 4)^5$

53. $(a + 3b)^4$

54. $(4p - q)^6$

55. $(w^3 - 3)^4$

56. $(2s^4 + 5)^5$

57. $(3u + v^2)^6$

58. $(x^3 - y^2)^4$

In Exercises 59–66, use the given value of n to find the coefficient of x^n in the expansion of the binomial. *(See Example 7b.)*

59. $(x - 2)^{10}, n = 5$

60. $(x - 3)^7, n = 4$

61. $(x^2 - 3)^8, n = 6$

62. $(3x + 2)^5, n = 3$

63. $(2x + 5)^{12}, n = 7$

64. $(3x - 1)^9, n = 2$

65. $\left(\frac{1}{2}x - 4\right)^{11}, n = 4$

66. $\left(\frac{1}{4}x + 6\right)^6, n = 3$

67. **REASONING** Write the eighth row of Pascal's Triangle as combinations and as numbers.

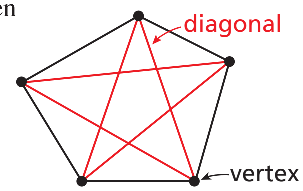
68. **PROBLEM SOLVING** The first four triangular numbers are 1, 3, 6, and 10.

a. Use Pascal's Triangle to write the first four triangular numbers as combinations.

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1
\end{array}
$$

b. Use your result from part (a) to write an explicit rule for the nth triangular number T_n.

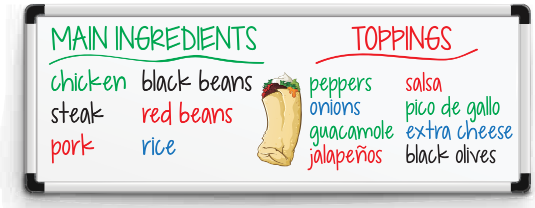
69. **MATHEMATICAL CONNECTIONS** A polygon is convex when no line that contains a side of the polygon contains a point in the interior of the polygon. Consider a convex polygon with n sides.



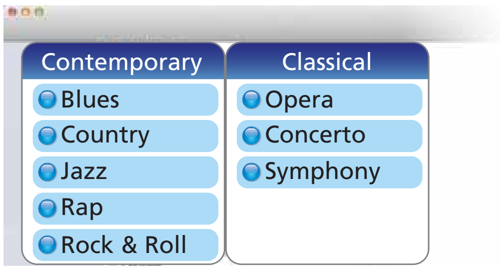
a. Use the combinations formula to write an expression for the number of diagonals in an n-sided polygon.

b. Use your result from part (a) to write a formula for the number of diagonals of an n-sided convex polygon.

70. **PROBLEM SOLVING** You are ordering a burrito with 2 main ingredients and 3 toppings. The menu below shows the possible choices. How many different burritos are possible?



71. **PROBLEM SOLVING** You want to purchase 2 different types of contemporary music CDs and 1 classical music CD from the music collection shown. How many different sets of music types can you choose for your purchase?



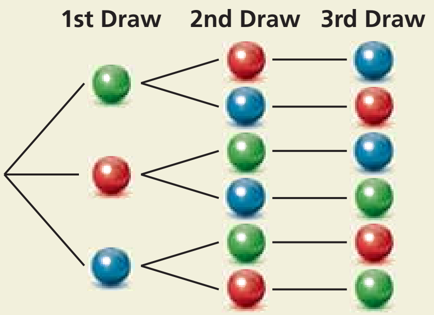
72. **PROBLEM SOLVING** Every student in your history class is required to present a project in front of the class. Each day, 4 students make their presentations in an order chosen at random by the teacher. You make your presentation on the first day.

a. What is the probability that you are chosen to be the first or second presenter on the first day?

b. What is the probability that you are chosen to be the second or third presenter on the first day? Compare your answer with that in part (a).

73. PROBLEM SOLVING The organizer of a cast party for a drama club asks each of the 6 cast members to bring 1 food item from a list of 10 items. Assuming each member randomly chooses a food item to bring, what is the probability that at least 2 of the 6 cast members bring the same item?

74. HOW DO YOU SEE IT? A bag contains one green marble, one red marble, and one blue marble. The diagram shows the possible outcomes of randomly drawing three marbles from the bag without replacement.



a. How many combinations of three marbles can be drawn from the bag? Explain.

b. How many permutations of three marbles can be drawn from the bag? Explain.

75. PROBLEM SOLVING You are one of 10 students performing in a school talent show. The order of the performances is determined at random. The first 5 performers go on stage before the intermission.

a. What is the probability that you are the last performer before the intermission and your rival performs immediately before you?

b. What is the probability that you are *not* the first performer?

76. THOUGHT PROVOKING How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3, and 4? Repetition of digits is allowed.

77. PROBLEM SOLVING Consider a standard deck of 52 playing cards. The order in which the cards are dealt for a "hand" does not matter.

a. How many different 5-card hands are possible?

b. How many different 5-card hands have all 5 cards of a single suit?

78. PROBLEM SOLVING There are 30 students in your class. Your science teacher chooses 5 students at random to complete a group project. Find the probability that you and your 2 best friends in the science class are chosen to work in the group. Explain how you found your answer.

79. PROBLEM SOLVING Follow the steps below to explore a famous probability problem called the *birthday problem*. (Assume there are 365 equally likely birthdays possible.)

a. What is the probability that at least 2 people share the same birthday in a group of 6 randomly chosen people? in a group of 10 randomly chosen people?

b. Generalize the results from part (a) by writing a formula for the probability $P(n)$ that at least 2 people in a group of n people share the same birthday. (*Hint*: Use ${}_nP_r$ notation in your formula.)

c. Enter the formula from part (b) into a graphing calculator. Use the *table* feature to make a table of values. For what group size does the probability that at least 2 people share the same birthday first exceed 50%?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

80. A bag contains 12 white marbles and 3 black marbles. You pick 1 marble at random. What is the probability that you pick a black marble? *(Section 10.1)*

81. The table shows the result of flipping two coins 12 times. For what outcome is the experimental probability the same as the theoretical probability? *(Section 10.1)*

HH	HT	TH	TT
2	6	3	1