10.6 Binomial Distributions

Essential Question How can you determine the frequency of each outcome of an event?

EXPLORATION 1 Analyzing Histograms

Work with a partner. The histograms show the results when n coins are flipped.

STUDY TIP

When 4 coins are flipped ($n = 4$), the possible outcomes are

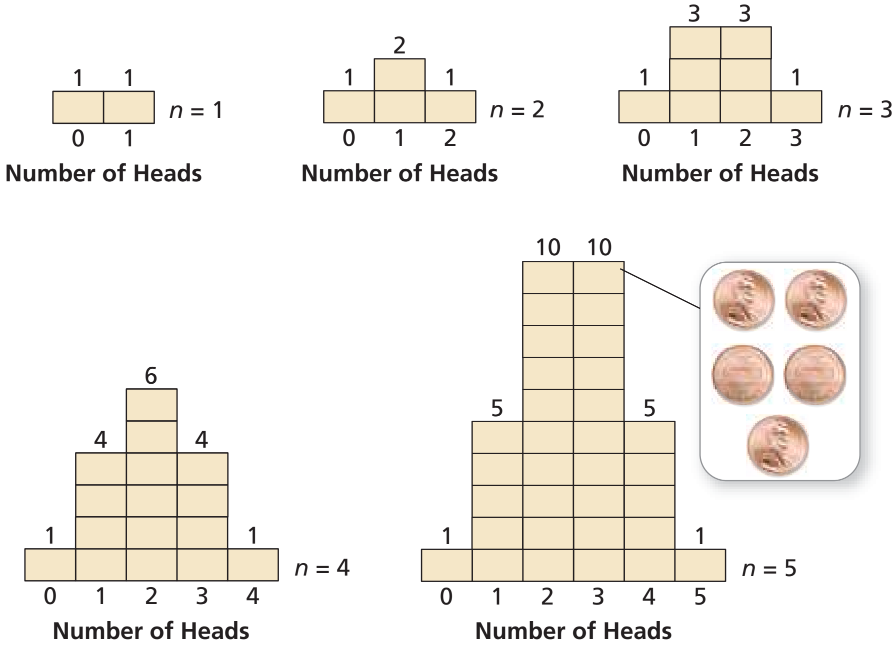
TTTT TTTH TTHT TTHH

THTT THTH THHT THHH

HTTT HTTH HTHT HTHH

HHTT HHTH HHHT HHHH.

The histogram shows the numbers of outcomes having 0, 1, 2, 3, and 4 heads.



a. In how many ways can 3 heads occur when 5 coins are flipped?

b. Draw a histogram that shows the numbers of heads that can occur when 6 coins are flipped.

c. In how many ways can 3 heads occur when 6 coins are flipped?

EXPLORATION 2 Determining the Number of Occurrences

Work with a partner.

LOOKING FOR A PATTERN

To be proficient in math, you need to look closely to discern a pattern or structure.

a. Complete the table showing the numbers of ways in which 2 heads can occur when n coins are flipped.

n	3	4	5	6	7
Occurrences of 2 heads					

b. Determine the pattern shown in the table. Use your result to find the number of ways in which 2 heads can occur when 8 coins are flipped.

Communicate Your Answer

3. How can you determine the frequency of each outcome of an event?

4. How can you use a histogram to find the probability of an event?

10.6 Lesson

Core Vocabulary

random variable, *p. 580*
probability distribution, *p. 580*
binomial distribution, *p. 581*
binomial experiment, *p. 581*

Previous
histogram

What You Will Learn

- Construct and interpret probability distributions.
- Construct and interpret binomial distributions.

Probability Distributions

A **random variable** is a variable whose value is determined by the outcomes of a probability experiment. For example, when you roll a six-sided die, you can define a random variable x that represents the number showing on the die. So, the possible values of x are 1, 2, 3, 4, 5, and 6. For every random variable, a *probability distribution* can be defined.

Core Concept

Probability Distributions

A **probability distribution** is a function that gives the probability of each possible value of a random variable. The sum of all the probabilities in a probability distribution must equal 1.

Probability Distribution for Rolling a Six-Sided Die

x	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

EXAMPLE 1 Constructing a Probability Distribution

Let x be a random variable that represents the sum when two six-sided dice are rolled. Make a table and draw a histogram showing the probability distribution for x.

SOLUTION

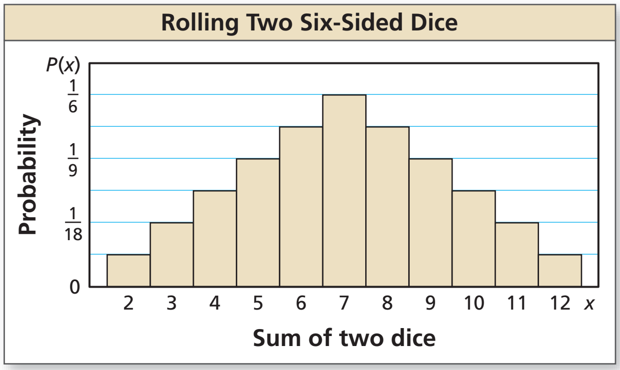
Step 1 Make a table. The possible values of x are the integers from 2 to 12. The table shows how many outcomes of rolling two dice produce each value of x. Divide the number of outcomes for x by 36 to find $P(x)$.

x (sum)	2	3	4	5	6	7	8	9	10	11	12
Outcomes	1	2	3	4	5	6	5	4	3	2	1
$P(x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

STUDY TIP

Recall that there are 36 possible outcomes when rolling two six-sided dice. These are listed in Example 3 on page 540.

Step 2 Draw a histogram where the intervals are given by x and the frequencies are given by $P(x)$.



EXAMPLE 2 Interpreting a Probability Distribution

Use the probability distribution in Example 1 to answer each question.

a. What is the most likely sum when rolling two six-sided dice?

b. What is the probability that the sum of the two dice is at least 10?

SOLUTION

a. The most likely sum when rolling two six-sided dice is the value of x for which $P(x)$ is greatest. This probability is greatest for $x = 7$. So, when rolling the two dice, the most likely sum is 7.

b. The probability that the sum of the two dice is at least 10 is

$$\begin{aligned} P(x \geq 10) &= P(x = 10) + P(x = 11) + P(x = 12) \\ &= \frac{3}{36} + \frac{2}{36} + \frac{1}{36} \\ &= \frac{6}{36} \\ &= \frac{1}{6} \\ &\approx 0.167. \end{aligned}$$

▶ The probability is about 16.7%.

Monitoring Progress

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An octahedral die has eight sides numbered 1 through 8. Let x be a random variable that represents the sum when two such dice are rolled.

1. Make a table and draw a histogram showing the probability distribution for x.

2. What is the most likely sum when rolling the two dice?

3. What is the probability that the sum of the two dice is at most 3?

Binomial Distributions

One type of probability distribution is a **binomial distribution**. A binomial distribution shows the probabilities of the outcomes of a *binomial experiment*.

Core Concept

Binomial Experiments

A **binomial experiment** meets the following conditions.

- There are n independent trials.
- Each trial has only two possible outcomes: success and failure.
- The probability of success is the same for each trial. This probability is denoted by p. The probability of failure is $1 - p$.

For a binomial experiment, the probability of exactly k successes in n trials is

$$P(k \text{ successes}) = {}_nC_k p^k (1 - p)^{n-k}.$$

EXAMPLE 3 Constructing a Binomial Distribution

According to a survey, about 33% of people ages 16 and older in the U.S. own an electronic book reading device, or e-reader. You ask 6 randomly chosen people (ages 16 and older) whether they own an e-reader. Draw a histogram of the binomial distribution for your survey.

ATTENDING TO PRECISION

When probabilities are rounded, the sum of the probabilities may differ slightly from 1.

SOLUTION

The probability that a randomly selected person has an e-reader is $p = 0.33$. Because you survey 6 people, $n = 6$.

$P(k = 0) = {}_6C_0(0.33)^0(0.67)^6 \approx 0.090$

$P(k = 1) = {}_6C_1(0.33)^1(0.67)^5 \approx 0.267$

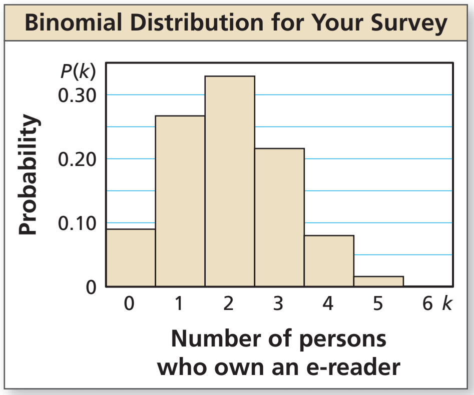
$P(k = 2) = {}_6C_2(0.33)^2(0.67)^4 \approx 0.329$

$P(k = 3) = {}_6C_3(0.33)^3(0.67)^3 \approx 0.216$

$P(k = 4) = {}_6C_4(0.33)^4(0.67)^2 \approx 0.080$

$P(k = 5) = {}_6C_5(0.33)^5(0.67)^1 \approx 0.016$

$P(k = 6) = {}_6C_6(0.33)^6(0.67)^0 \approx 0.001$



A histogram of the distribution is shown.

EXAMPLE 4 Interpreting a Binomial Distribution

Use the binomial distribution in Example 3 to answer each question.

a. What is the most likely outcome of the survey?

b. What is the probability that at most 2 people have an e-reader?

COMMON ERROR

Because a person may not have an e-reader, be sure you include $P(k = 0)$ when finding the probability that at most 2 people have an e-reader.

SOLUTION

a. The most likely outcome of the survey is the value of k for which $P(k)$ is greatest. This probability is greatest for $k = 2$. The most likely outcome is that 2 of the 6 people own an e-reader.

b. The probability that at most 2 people have an e-reader is

$$P(k \leq 2) = P(k = 0) + P(k = 1) + P(k = 2)$$
$$\approx 0.090 + 0.267 + 0.329$$
$$\approx 0.686.$$

▶ The probability is about 68.6%.

Monitoring Progress

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According to a survey, about 85% of people ages 18 and older in the U.S. use the Internet or e-mail. You ask 4 randomly chosen people (ages 18 and older) whether they use the Internet or e-mail.

4. Draw a histogram of the binomial distribution for your survey.

5. What is the most likely outcome of your survey?

6. What is the probability that at most 2 people you survey use the Internet or e-mail?

10.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What is a random variable?

2. **WRITING** Give an example of a binomial experiment and describe how it meets the conditions of a binomial experiment.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, make a table and draw a histogram showing the probability distribution for the random variable. *(See Example 1.)*

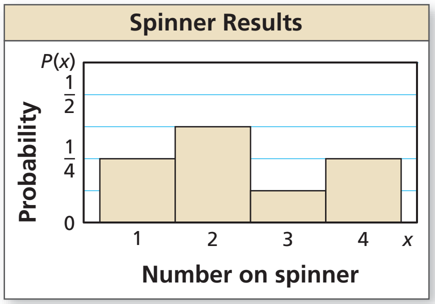
3. x = the number on a table tennis ball randomly chosen from a bag that contains 5 balls labeled "1," 3 balls labeled "2," and 2 balls labeled "3."

4. $c = 1$ when a randomly chosen card out of a standard deck of 52 playing cards is a heart and $c = 2$ otherwise.

5. $w = 1$ when a randomly chosen letter from the English alphabet is a vowel and $w = 2$ otherwise.

6. n = the number of digits in a random integer from 0 through 999.

In Exercises 7 and 8, use the probability distribution to determine (a) the number that is most likely to be spun on a spinner, and (b) the probability of spinning an even number. *(See Example 2.)*

7.


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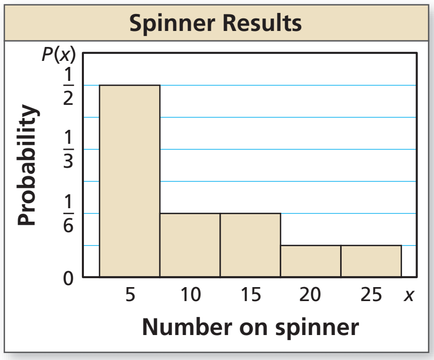


USING EQUATIONS In Exercises 9–12, calculate the probability of flipping a coin 20 times and getting the given number of heads.

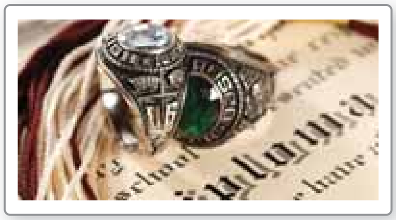
9. 1

10. 4

11. 18

12. 20

13. **MODELING WITH MATHEMATICS** According to a survey, 27% of high school students in the United States buy a class ring. You ask 6 randomly chosen high school students whether they own a class ring. *(See Examples 3 and 4.)*



a. Draw a histogram of the binomial distribution for your survey.

b. What is the most likely outcome of your survey?

c. What is the probability that at most 2 people have a class ring?

14. **MODELING WITH MATHEMATICS** According to a survey, 48% of adults in the United States believe that Unidentified Flying Objects (UFOs) are observing our planet. You ask 8 randomly chosen adults whether they believe UFOs are watching Earth.

a. Draw a histogram of the binomial distribution for your survey.

b. What is the most likely outcome of your survey?

c. What is the probability that at most 3 people believe UFOs are watching Earth?

ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in calculating the probability of rolling a 1 exactly 3 times in 5 rolls of a six-sided die.**

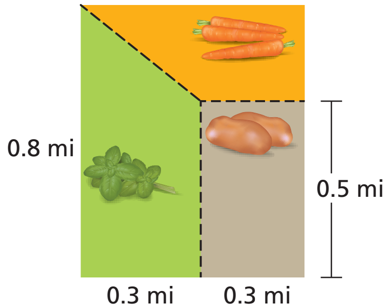
15. ✗

$$P(k = 3) = {}_5C_3\left(\frac{1}{6}\right)^{5-3}\left(\frac{5}{6}\right)^3$$
$$\approx 0.161$$

16. ✗

$$P(k = 3) = \left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{5-3}$$
$$\approx 0.003$$

17. MATHEMATICAL CONNECTIONS At most 7 gopher holes appear each week on the farm shown. Let x represent how many of the gopher holes appear in the carrot patch. Assume that a gopher hole has an equal chance of appearing at any point on the farm.



a. Find $P(x)$ for $x = 0, 1, 2, \ldots, 7$.

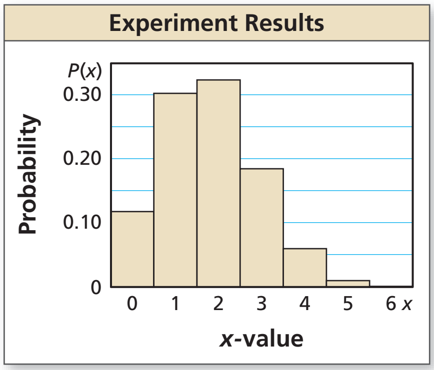
b. Make a table showing the probability distribution for x.

c. Make a histogram showing the probability distribution for x.

18. HOW DO YOU SEE IT? Complete the probability distribution for the random variable x. What is the probability the value of x is greater than 2?

x	1	2	3	4
$P(x)$	0.1	0.3	0.4	

19. MAKING AN ARGUMENT The binomial distribution shows the results of a binomial experiment. Your friend claims that the probability p of a success must be greater than the probability $1 - p$ of a failure. Is your friend correct? Explain your reasoning.



20. THOUGHT PROVOKING There are 100 coins in a bag. Only one of them has a date of 2010. You choose a coin at random, check the date, and then put the coin back in the bag. You repeat this 100 times. Are you certain of choosing the 2010 coin at least once? Explain your reasoning.

21. MODELING WITH MATHEMATICS Assume that having a male and having a female child are independent events, and that the probability of each is 0.5.

a. A couple has 4 male children. Evaluate the validity of this statement: "The first 4 kids were all boys, so the next one will probably be a girl."

b. What is the probability of having 4 male children and then a female child?

c. Let x be a random variable that represents the number of children a couple already has when they have their first female child. Draw a histogram of the distribution of $P(x)$ for $0 \le x \le 10$. Describe the shape of the histogram.

22. CRITICAL THINKING An entertainment system has n speakers. Each speaker will function properly with probability p, independent of whether the other speakers are functioning. The system will operate effectively when at least 50% of its speakers are functioning. For what values of p is a 5-speaker system more likely to operate than a 3-speaker system?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

List the possible outcomes for the situation. *(Section 10.1)*

23. guessing the gender of three children

24. picking one of two doors and one of three curtains