11.4 Experimental Design

Essential Question How can you use an experiment to test a conjecture?

EXPLORATION 1 Using an Experiment

Work with a partner. Standard white playing dice are manufactured with black dots that are indentations, as shown. So, the side with six indentations is the lightest side and the side with one indentation is the heaviest side.

You make a conjecture that when you roll a standard playing die, the number 6 will come up more often than the number 1 because 6 is the lightest side. To test your conjecture, roll a standard playing die 25 times. Record the results in the table. Does the experiment confirm your conjecture? Explain your reasoning.

Number						
Rolls						

EXPLORATION 2 Analyzing an Experiment

Work with a partner. To overcome the imbalance of standard playing dice, one of the authors of this book invented and patented 12-sided dice, on which each number from 1 through 6 appears twice (on opposing sides). See *BigIdeasMath.com*.

As part of the patent process, a standard playing die was rolled 27,090 times. The results are shown below.

Number	1	2	3	4	5	6
Rolls	4293	4524	4492	4397	4623	4761

What can you conclude from the results of this experiment? Explain your reasoning.

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and perform experiments to explore the truth of your conjectures.

Communicate Your Answer

3. How can you use an experiment to test a conjecture?
4. Exploration 2 shows the results of rolling a standard playing die 27,090 times to test the conjecture in Exploration 1. Why do you think the number of trials was so large?
5. Make a conjecture about the outcomes of rolling the 12-sided die in Exploration 2. Then design an experiment that could be used to test your conjecture. Be sure that your experiment is practical to complete and includes enough trials to give meaningful results.

11.4 Lesson

Core Vocabulary

controlled experiment, *p. 620*
control group, *p. 620*
treatment group, *p. 620*
randomization, *p. 620*
randomized comparative experiment, *p. 620*
placebo, *p. 620*
replication, *p. 622*

Previous
sample size

What You Will Learn

- Describe experiments.
- Recognize how randomization applies to experiments and observational studies.
- Analyze experimental designs.

Describing Experiments

In a **controlled experiment**, two groups are studied under identical conditions with the exception of one variable. The group under ordinary conditions that is subjected to no treatment is the **control group**. The group that is subjected to the treatment is the **treatment group**.

Randomization is a process of randomly assigning subjects to different treatment groups. In a **randomized comparative experiment**, subjects are randomly assigned to the control group or the treatment group. In some cases, subjects in the control group are given a **placebo**, which is a harmless, unmedicated treatment that resembles the actual treatment. The comparison of the control group and the treatment group makes it possible to determine any effects of the treatment.

Randomization minimizes bias and produces groups of individuals who are theoretically similar in all ways before the treatment is applied. Conclusions drawn from an experiment that is not a randomized comparative experiment may not be valid.

EXAMPLE 1 Evaluating Published Reports

Determine whether each study is a randomized comparative experiment. If it is, describe the treatment, the treatment group, and the control group. If it is not, explain why not and discuss whether the conclusions drawn from the study are valid.

a.

Health Watch

Vitamin C Lowers Cholesterol

At a health clinic, patients were given the choice of whether to take a dietary supplement of 500 milligrams of vitamin C each day. Fifty patients who took the supplement were monitored for one year, as were 50 patients who did not take the supplement. At the end of one year, patients who took the supplement had 15% lower cholesterol levels than patients in the other group.

b.

Supermarket Checkout

Check Out Even Faster

To test the new design of its self checkout, a grocer gathered 142 customers and randomly divided them into two groups. One group used the new self checkout and one group used the old self checkout to buy the same groceries. Users of the new self checkout were able to complete their purchases 16% faster.

STUDY TIP

The study in part (a) is an *observational study* because the treatment is not being imposed.

SOLUTION

a. The study is not a randomized comparative experiment because the individuals were not randomly assigned to a control group and a treatment group. The conclusion that vitamin C lowers cholesterol may or may not be valid. There may be other reasons why patients who took the supplement had lower cholesterol levels. For instance, patients who voluntarily take the supplement may be more likely to have other healthy eating or lifestyle habits that could affect their cholesterol levels.

b. The study is a randomized comparative experiment. The treatment is the use of the new self checkout. The treatment group is the individuals who use the new self checkout. The control group is the individuals who use the old self checkout.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. Determine whether the study is a randomized comparative experiment. If it is, describe the treatment, the treatment group, and the control group. If it is not, explain why not and discuss whether the conclusions drawn from the study are valid.

Motorist News

Early Birds Make Better Drivers

A recent study shows that adults who rise before 6:30 A.M. are better drivers than other adults. The study monitored the driving records of 140 volunteers who always wake up before 6:30 and 140 volunteers who never wake up before 6:30. The early risers had 12% fewer accidents.

Randomization in Experiments and Observational Studies

You have already learned about random sampling and its usefulness in surveys. Randomization applies to experiments and observational studies as shown below.

Experiment	Observational study
Individuals are assigned at random to the treatment group or the control group.	When possible, random samples can be selected for the groups being studied.

Good experiments and observational studies are designed to compare data from two or more groups and to show any relationship between variables. Only a well-designed *experiment*, however, can determine a cause-and-effect relationship.

Core Concept

Comparative Studies and Causality

- A rigorous randomized comparative experiment, by eliminating sources of variation other than the controlled variable, can make valid cause-and-effect conclusions possible.
- An observational study can identify *correlation* between variables, but not *causality*. Variables, other than what is being measured, may be affecting the results.

EXAMPLE 2 Designing an Experiment or Observational Study

Explain whether the following research topic is best investigated through an experiment or an observational study. Then describe the design of the experiment or observational study.

You want to know whether vigorous exercise in older people results in longer life.

SOLUTION

The treatment, vigorous exercise, is not possible for those people who are already unhealthy, so it is not ethical to assign individuals to a control or treatment group. Use an observational study. Randomly choose one group of individuals who already exercise vigorously. Then randomly choose one group of individuals who do not exercise vigorously. Monitor the ages of the individuals in both groups at regular intervals. Note that because you are using an observational study, you should be able to identify a *correlation* between vigorous exercise in older people and longevity, but not *causality*.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

2. Determine whether the following research topic is best investigated through an experiment or an observational study. Then describe the design of the experiment or observational study.

   You want to know whether flowers sprayed twice per day with a mist of water stay fresh longer than flowers that are not sprayed.

Analyzing Experimental Designs

An important part of experimental design is *sample size*, or the number of subjects in the experiment. To improve the validity of the experiment, **replication** is required, which is repetition of the experiment under the same or similar conditions.

UNDERSTANDING MATHEMATICAL TERMS

The *validity* of an experiment refers to the reliability of the results. The results of a valid experiment are more likely to be accepted.

EXAMPLE 3 Analyzing Experimental Designs

A pharmaceutical company wants to test the effectiveness of a new chewing gum designed to help people lose weight. Identify a potential problem, if any, with each experimental design. Then describe how you can improve it.

a. The company identifies 10 people who are overweight. Five subjects are given the new chewing gum and the other 5 are given a placebo. After 3 months, each subject is evaluated and it is determined that the 5 subjects who have been using the new chewing gum have lost weight.

b. The company identifies 10,000 people who are overweight. The subjects are divided into groups according to gender. Females receive the new chewing gum and males receive the placebo. After 3 months, a significantly large number of the female subjects have lost weight.

c. The company identifies 10,000 people who are overweight. The subjects are divided into groups according to age. Within each age group, subjects are randomly assigned to receive the new chewing gum or the placebo. After 3 months, a significantly large number of the subjects who received the new chewing gum have lost weight.

STUDY TIP

The experimental design described in part (c) is an example of *randomized block design*.

SOLUTION

a. The sample size is not large enough to produce valid results. To improve the validity of the experiment, the sample size must be larger and the experiment must be replicated.

b. Because the subjects are divided into groups according to gender, the groups are not similar. The new chewing gum may have more of an effect on women than on men, or more of an effect on men than on women. It is not possible to see such an effect with the experiment the way it is designed. The subjects can be divided into groups according to gender, but within each group, they must be randomly assigned to the treatment group or the control group.

c. The subjects are divided into groups according to a similar characteristic (age). Because subjects within each age group are randomly assigned to receive the new chewing gum or the placebo, replication is possible.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

3. In Example 3, the company identifies 250 people who are overweight. The subjects are randomly assigned to a treatment group or a control group. In addition, each subject is given a DVD that documents the dangers of obesity. After 3 months, most of the subjects placed in the treatment group have lost weight. Identify a potential problem with the experimental design. Then describe how you can improve it.

4. You design an experiment to test the effectiveness of a vaccine against a strain of influenza. In the experiment, 100,000 people receive the vaccine and another 100,000 people receive a placebo. Identify a potential problem with the experimental design. Then describe how you can improve it.

11.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Repetition of an experiment under the same or similar conditions is called _____.

2. **WRITING** Describe the difference between the control group and the treatment group in a controlled experiment.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, determine whether the study is a randomized comparative experiment. If it is, describe the treatment, the treatment group, and the control group. If it is not, explain why not and discuss whether the conclusions drawn from the study are valid. *(See Example 1.)*

3. ***Insomnia***

New Drug Improves Sleep

To test a new drug for insomnia, a pharmaceutical company randomly divided 200 adult volunteers into two groups. One group received the drug and one group received a placebo. After one month, the adults who took the drug slept 18% longer, while those who took the placebo experienced no significant change.

4. ***Dental Health***

Milk Fights Cavities

At a middle school, students can choose to drink milk or other beverages at lunch. Seventy-five students who chose milk were monitored for one year, as were 75 students who chose other beverages. At the end of the year, students in the "milk" group had 25% fewer cavities than students in the other group.

ERROR ANALYSIS In Exercises 5 and 6, describe and correct the error in describing the study.

A company's researchers want to study the effects of adding shea butter to their existing hair conditioner. They monitor the hair quality of 30 randomly selected customers using the regular conditioner and 30 randomly selected customers using the new shea butter conditioner.

5. ✗ The control group is individuals who do not use either of the conditioners.

6. ✗ The study is an observational study.

In Exercises 7–10, explain whether the research topic is best investigated through an experiment or an observational study. Then describe the design of the experiment or observational study. *(See Example 2.)*

7. A researcher wants to compare the body mass index of smokers and nonsmokers.

8. A restaurant chef wants to know which pasta sauce recipe is preferred by more diners.

9. A farmer wants to know whether a new fertilizer affects the weight of the fruit produced by strawberry plants.

10. You want to know whether homes that are close to parks or schools have higher property values.

11. **DRAWING CONCLUSIONS** A company wants to test whether a nutritional supplement has an adverse effect on an athlete's heart rate while exercising. Identify a potential problem, if any, with each experimental design. Then describe how you can improve it. *(See Example 3.)*

    a. The company randomly selects 250 athletes. Half of the athletes receive the supplement and their heart rates are monitored while they run on a treadmill. The other half of the athletes are given a placebo and their heart rates are monitored while they lift weights. The heart rates of the athletes who took the supplement significantly increased while exercising.

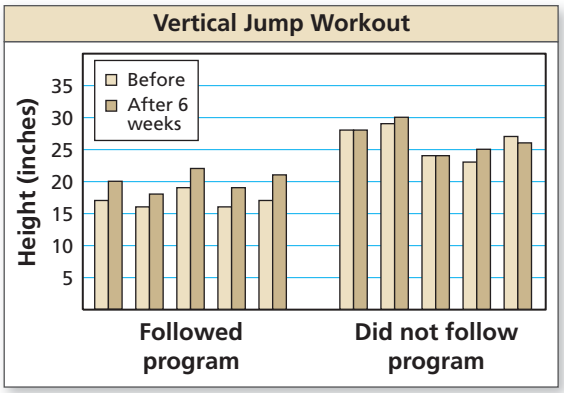
    b. The company selects 1000 athletes. The athletes are divided into two groups based on age. Within each age group, the athletes are randomly assigned to receive the supplement or the placebo. The athletes' heart rates are monitored while they run on a treadmill. There was no significant difference in the increases in heart rates between the two groups.

12. DRAWING CONCLUSIONS A researcher wants to test the effectiveness of reading novels on raising intelligence quotient (IQ) scores. Identify a potential problem, if any, with each experimental design. Then describe how you can improve it.

a. The researcher selects 500 adults and randomly divides them into two groups. One group reads novels daily and one group does not read novels. At the end of 1 year, each adult is evaluated and it is determined that neither group had an increase in IQ scores.

b. Fifty adults volunteer to spend time reading novels every day for 1 year. Fifty other adults volunteer to refrain from reading novels for 1 year. Each adult is evaluated and it is determined that the adults who read novels raised their IQ scores by 3 points more than the other group.

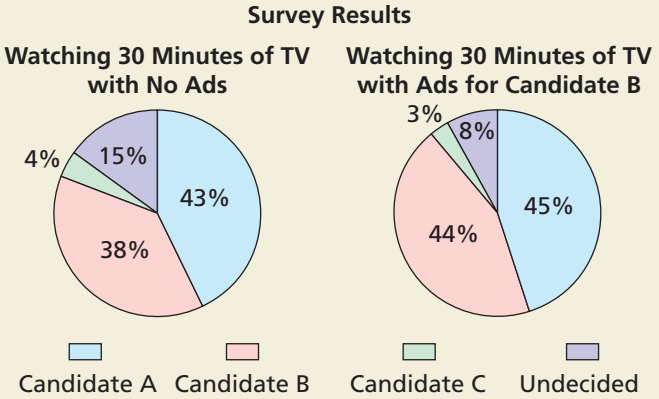
13. DRAWING CONCLUSIONS A fitness company claims that its workout program will increase vertical jump heights in 6 weeks. To test the workout program, 10 athletes are divided into two groups. The double bar graph shows the results of the experiment. Identify the potential problems with the experimental design. Then describe how you can improve it.



14. WRITING Explain why observational studies, rather than experiments, are usually used in astronomy.

15. MAKING AN ARGUMENT Your friend wants to determine whether the number of siblings has an effect on a student's grades. Your friend claims to be able to show causality between the number of siblings and grades. Is your friend correct? Explain.

16. HOW DO YOU SEE IT? To test the effect political advertisements have on voter preferences, a researcher selects 400 potential voters and randomly divides them into two groups. The circle graphs show the results of the study.



a. Is the study a randomized comparative experiment? Explain.

b. Describe the treatment.

c. Can you conclude that the political advertisements were effective? Explain.

17. WRITING Describe the *placebo effect* and how it affects the results of an experiment. Explain how a researcher can minimize the placebo effect.

18. THOUGHT PROVOKING Make a hypothesis about something that interests you. Design an experiment that could show that your hypothesis is probably true.

19. REASONING Will replicating an experiment on many individuals produce data that are more likely to accurately represent a population than performing the experiment only once? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Draw a dot plot that represents the data. Identify the shape of the distribution. *(Skills Review Handbook)*

20. Ages: 24, 21, 22, 26, 22, 23, 25, 23, 23, 24, 20, 25

21. Golf strokes: 4, 3, 4, 3, 3, 2, 7, 5, 3, 4

Tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function. *(Section 6.1)*

22. $y = 4^x$

23. $y = (0.95)^x$

24. $y = (0.2)^x$

25. $y = (1.25)^x$