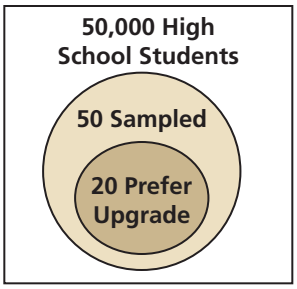
# 11.5 Making Inferences from Sample Surveys

**Essential Question** How can you use a sample survey to infer a conclusion about a population?

## EXPLORATION 1 Making an Inference from a Sample

**Work with a partner.** You conduct a study to determine what percent of the high school students in your city would prefer an upgraded model of their current cell phone. Based on your intuition and talking with a few acquaintances, you think that 50% of high school students would prefer an upgrade. You survey 50 randomly chosen high school students and find that 20 of them prefer an upgraded model.



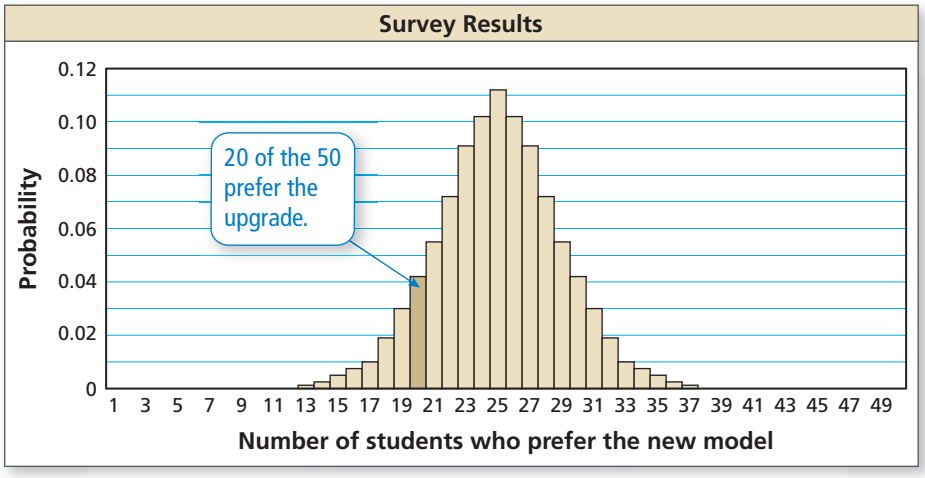

**MODELING WITH MATHEMATICS**

To be proficient in math, you need to apply the mathematics you know to solve problems arising in everyday life.

**a.** Based on your sample survey, what percent of the high school students in your city would prefer an upgraded model? Explain your reasoning.

**b.** In spite of your sample survey, is it still possible that 50% of the high school students in your city prefer an upgraded model? Explain your reasoning.

**c.** To investigate the likelihood that you could have selected a sample of 50 from a population in which 50% of the population does prefer an upgraded model, you create a binomial distribution as shown below. From the distribution, estimate the probability that exactly 20 students surveyed prefer an upgraded model. Is this event likely to occur? Explain your reasoning.




**d.** When making inferences from sample surveys, the sample must be random. In the situation described above, describe how you could design and conduct a survey using a random sample of 50 high school students who live in a large city.

## Communicate Your Answer

**2.** How can you use a sample survey to infer a conclusion about a population?

**3.** In Exploration 1(c), what is the probability that exactly 25 students you survey prefer an upgraded model?

# 11.5 Lesson

## Core Vocabulary

descriptive statistics, *p. 626*
inferential statistics, *p. 626*
margin of error, *p. 629*

***Previous***
statistic
parameter

## What You Will Learn

- Estimate population parameters.
- Analyze estimated population parameters.
- Find margins of error for surveys.

## Estimating Population Parameters

The study of statistics has two major branches: *descriptive statistics* and *inferential statistics*. **Descriptive statistics** involves the organization, summarization, and display of data. So far, you have been using descriptive statistics in your studies of data analysis and statistics. **Inferential statistics** involves using a sample to draw conclusions about a population. You can use statistics to make reasonable predictions, or *inferences*, about an entire population when the sample is representative of the population.

### EXAMPLE 1 Estimating a Population Mean

The numbers of friends for a random sample of 40 teen users of a social networking website are shown in the table. Estimate the population mean $\mu$.

| Number of Friends | | | | |
|---|---|---|---|---|
| 281 | 342 | 229 | 384 | 320 |
| 247 | 298 | 248 | 312 | 445 |
| 385 | 286 | 314 | 260 | 186 |
| 287 | 342 | 225 | 308 | 343 |
| 262 | 220 | 320 | 310 | 150 |
| 274 | 291 | 300 | 410 | 255 |
| 279 | 351 | 370 | 257 | 350 |
| 369 | 215 | 325 | 338 | 278 |

**REMEMBER**

Recall that $\bar{x}$ denotes the sample mean. It is read as "$x$ bar."

**SOLUTION**

To estimate the unknown population mean $\mu$, find the sample mean $\bar{x}$.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{11{,}966}{40} = 299.15$$

- So, the mean number of friends for all teen users of the website is about 299.

**STUDY TIP**

The probability that the population mean is *exactly* 299.15 is virtually 0, but the sample mean is a good estimate of $\mu$.

## Monitoring Progress

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1. The data from another random sample of 30 teen users of the social networking website are shown in the table. Estimate the population mean $\mu$.

| Number of Friends | | | | |
|---|---|---|---|---|
| 305 | 237 | 261 | 374 | 341 |
| 257 | 243 | 352 | 330 | 189 |
| 297 | 418 | 275 | 288 | 307 |
| 295 | 288 | 341 | 322 | 271 |
| 209 | 164 | 363 | 228 | 390 |
| 313 | 315 | 263 | 299 | 285 |

Not every random sample results in the same estimate of a population parameter; there will be some sampling variability. Larger sample sizes, however, tend to produce more accurate estimates.

**REMEMBER**

A *population proportion* is the ratio of members of a population with a particular characteristic to the total members of the population. A *sample proportion* is the ratio of members of a sample of the population with a particular characteristic to the total members of the sample.

## EXAMPLE 2 Estimating Population Proportions

A student newspaper wants to predict the winner of a city's mayoral election. Two candidates, A and B, are running for office. Eight staff members conduct surveys of randomly selected residents. The residents are asked whether they will vote for Candidate A. The results are shown in the table.

| Sample Size | Number of Votes for Candidate A in the Sample | Percent of Votes for Candidate A in the Sample |
|---|---|---|
| 5 | 2 | 40% |
| 12 | 4 | 33.3% |
| 20 | 12 | 60% |
| 30 | 17 | 56.7% |
| 50 | 29 | 58% |
| 125 | 73 | 58.4% |
| 150 | 88 | 58.7% |
| 200 | 118 | 59% |

**a.** Based on the results of the first two sample surveys, do you think Candidate A will win the election? Explain.

**b.** Based on the results in the table, do you think Candidate A will win the election? Explain.

**STUDY TIP**

Statistics and probability provide information that you can use to weigh evidence and make decisions.

### SOLUTION

**a.** The results of the first two surveys (sizes 5 and 12) show that fewer than 50% of the residents will vote for Candidate A. Because there are only two candidates, one candidate needs more than 50% of the votes to win.

▶ Based on these surveys, you can predict Candidate A will not win the election.

**b.** As the sample sizes increase, the estimated percent of votes approaches 59%. You can predict that 59% of the city residents will vote for Candidate A.

▶ Because 59% of the votes are more than the 50% needed to win, you should feel confident that Candidate A will win the election.

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**2.** Two candidates are running for class president. The table shows the results of four surveys of random students in the class. The students were asked whether they will vote for the incumbent. Do you think the incumbent will be reelected? Explain.

| Sample Size | Number of "Yes" Responses | Percent of Votes for Incumbent |
|---|---|---|
| 10 | 7 | 70% |
| 20 | 11 | 55% |
| 30 | 13 | 43.3% |
| 40 | 17 | 42.5% |

## Analyzing Estimated Population Parameters

An estimated population parameter is a hypothesis. You learned in Section 11.2 that one way to analyze a hypothesis is to perform a simulation.

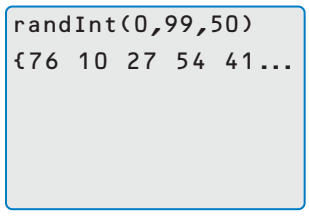
### EXAMPLE 3 Analyzing an Estimated Population Proportion

A national polling company claims 34% of U.S. adults say mathematics is the most valuable school subject in their lives. You survey a random sample of 50 adults.

**a.** What can you conclude about the accuracy of the claim that the population proportion is 0.34 when 15 adults in your survey say mathematics is the most valuable subject?

**b.** What can you conclude about the accuracy of the claim when 25 adults in your survey say mathematics is the most valuable subject?

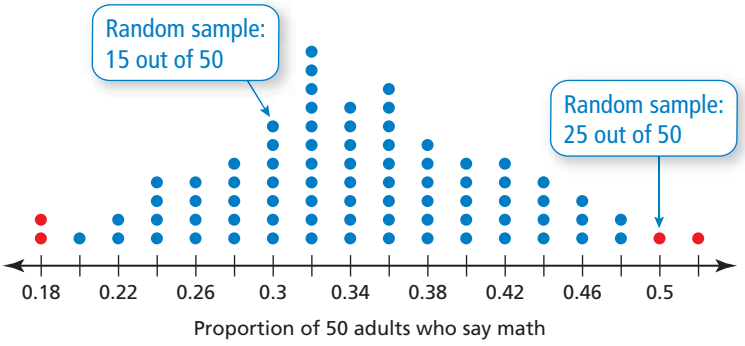
**c.** Assume that the true population proportion is 0.34. Estimate the variation among sample proportions using samples of size 50.




### SOLUTION

The polling company's claim (hypothesis) is that the population proportion of U.S. adults who say mathematics is the most valuable school subject is 0.34. To analyze this claim, simulate choosing 80 random samples of size 50 using a random number generator on a graphing calculator. Generate 50 random numbers from 0 to 99 for each sample. Let numbers 1 through 34 represent adults who say math. Find the sample proportions and make a dot plot showing the distribution of the sample proportions.

**Simulation: Polling 50 Adults**




**STUDY TIP**

The dot plot shows the results of one simulation. Results of other simulations may give slightly different results but the shape should be similar.

**a.** Note that 15 out of 50 corresponds to a sample proportion of $\frac{15}{50} = 0.3$. In the simulation, this result occurred in 7 of the 80 random samples. It is *likely* that 15 adults out of 50 would say math is the most valuable subject when the true population percentage is 34%. So, you can conclude the company's claim is probably accurate.

**b.** Note that 25 out of 50 corresponds to a sample proportion of $\frac{25}{50} = 0.5$. In the simulation, this result occurred in only 1 of the 80 random samples. So, it is *unlikely* that 25 adults out of 50 would say math is the most valuable subject when the true population percentage is 34%. So, you can conclude the company's claim is probably *not* accurate.

**INTERPRETING MATHEMATICAL RESULTS**

Note that the sample proportion 0.3 in part (a) lies in this interval, while the sample proportion 0.5 in part (b) falls outside this interval.

**c.** Note that the dot plot is fairly bell-shaped and symmetric, so the distribution is approximately normal. In a normal distribution, you know that about 95% of the possible sample proportions will lie within two standard deviations of 0.34. Excluding the two least and two greatest sample proportions, represented by red dots ● in the dot plot, leaves 76 of 80, or 95%, of the sample proportions. These 76 proportions range from 0.2 to 0.48. So, 95% of the time, a sample proportion should lie in the interval from 0.2 to 0.48.

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3. **WHAT IF?** In Example 3, what can you conclude about the accuracy of the claim that the population proportion is 0.34 when 21 adults in your random sample say mathematics is the most valuable subject?

## Finding Margins of Error for Surveys

When conducting a survey, you need to make the size of your sample large enough so that it accurately represents the population. As the sample size increases, the *margin of error* decreases.

The **margin of error** gives a limit on how much the responses of the sample would differ from the responses of the population. For example, if 40% of the people in a poll favor a new tax law, and the margin of error is ±4%, then it is likely that between 36% and 44% of the entire population favor a new tax law.

### Core Concept

#### Margin of Error Formula

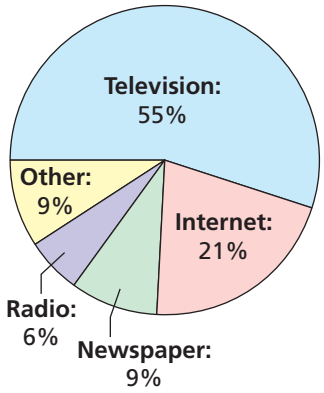
When a random sample of size $n$ is taken from a large population, the margin of error is approximated by

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}}.$$

This means that if the percent of the sample responding a certain way is $p$ (expressed as a decimal), then the percent of the population who would respond the same way is likely to be between $p - \frac{1}{\sqrt{n}}$ and $p + \frac{1}{\sqrt{n}}$.

### EXAMPLE 4 Finding a Margin of Error

Americans' Main News Source




In a survey of 2048 people in the U.S., 55% said that television is their main source of news. (a) What is the margin of error for the survey? (b) Give an interval that is likely to contain the exact percent of all people who use television as their main source of news.

#### SOLUTION

**a.** Use the margin of error formula.

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}} = \pm\frac{1}{\sqrt{2048}} \approx \pm 0.022$$

▶ The margin of error for the survey is about ±2.2%.

**b.** To find the interval, subtract and add 2.2% to the percent of people surveyed who said television is their main source of news (55%).

$$55\% - 2.2\% = 52.8\% \qquad 55\% + 2.2\% = 57.2\%$$

▶ It is likely that the exact percent of all people in the U.S. who use television as their main source of news is between 52.8% and 57.2%.

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4. In a survey of 1028 people in the U.S., 87% reported using the Internet. Give an interval that is likely to contain the exact percent of all people in the U.S. who use the Internet.

# 11.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The _____ gives a limit on how much the responses of the sample would differ from the responses of the population.

2. **WRITING** What is the difference between descriptive and inferential statistics?

## Monitoring Progress and Modeling with Mathematics

3. **PROBLEM SOLVING** The numbers of text messages sent each day by a random sample of 30 teen cellphone users are shown in the table. Estimate the population mean $\mu$. *(See Example 1.)*

| Number of Text Messages | | | | |
|---|---|---|---|---|
| 30 | 60 | 59 | 83 | 41 |
| 37 | 66 | 63 | 60 | 92 |
| 53 | 42 | 47 | 32 | 79 |
| 53 | 80 | 41 | 51 | 85 |
| 73 | 71 | 69 | 31 | 69 |
| 57 | 60 | 70 | 91 | 67 |

4. **PROBLEM SOLVING** The incomes for a random sample of 35 U.S. households are shown in the table. Estimate the population mean $\mu$.

| Income of U.S. Households | | | | |
|---|---|---|---|---|
| 14,300 | 52,100 | 74,800 | 51,000 | 91,500 |
| 72,800 | 50,500 | 15,000 | 37,600 | 22,100 |
| 40,000 | 65,400 | 50,000 | 81,100 | 99,800 |
| 43,300 | 32,500 | 76,300 | 83,400 | 24,600 |
| 30,800 | 62,100 | 32,800 | 21,900 | 64,400 |
| 73,100 | 20,000 | 49,700 | 71,000 | 45,900 |
| 53,200 | 45,500 | 55,300 | 19,100 | 63,100 |

5. **PROBLEM SOLVING** Use the data in Exercise 3 to answer each question.

   **a.** Estimate the population proportion $p$ of teen cellphone users who send more than 70 text messages each day.

   **b.** Estimate the population proportion $p$ of teen cellphone users who send fewer than 50 text messages each day.

6. **WRITING** A survey asks a random sample of U.S. teenagers how many hours of television they watch each night. The survey reveals that the sample mean is 3 hours per night. How confident are you that the average of all U.S. teenagers is exactly 3 hours per night? Explain your reasoning.

7. **DRAWING CONCLUSIONS** When the President of the United States vetoes a bill, the Congress can override the veto by a two-thirds majority vote in each House. Five news organizations conduct individual random surveys of U.S. Senators. The senators are asked whether they will vote to override the veto. The results are shown in the table. *(See Example 2.)*

| Sample Size | Number of Votes to Override Veto | Percent of Votes to Override Veto |
|---|---|---|
| 7 | 6 | 85.7% |
| 22 | 16 | 72.7% |
| 28 | 21 | 75% |
| 31 | 17 | 54.8% |
| 49 | 27 | 55.1% |

   **a.** Based on the results of the first two surveys, do you think the Senate will vote to override the veto? Explain.

   **b.** Based on the results in the table, do you think the Senate will vote to override the veto? Explain.

**8. DRAWING CONCLUSIONS** Your teacher lets the students decide whether to have their test on Friday or Monday. The table shows the results from four surveys of randomly selected students in your grade who are taking the same class. The students are asked whether they want to have the test on Friday.

| Sample Size | Number of "Yes" Responses | Percent of Votes |
|---|---|---|
| 10 | 8 | 80% |
| 20 | 12 | 60% |
| 30 | 16 | 53.3% |
| 40 | 18 | 45% |

**a.** Based on the results of the first two surveys, do you think the test will be on Friday? Explain.

**b.** Based on the results in the table, do you think the test will be on Friday? Explain.

**9. MODELING WITH MATHEMATICS** A national polling company claims that 54% of U.S. adults are married. You survey a random sample of 50 adults. *(See Example 3.)*

**a.** What can you conclude about the accuracy of the claim that the population proportion is 0.54 when 31 adults in your survey are married?

**b.** What can you conclude about the accuracy of the claim that the population proportion is 0.54 when 19 adults in your survey are married?

**c.** Assume that the true population proportion is 0.54. Estimate the variation among sample proportions for samples of size 50.

**10. MODELING WITH MATHEMATICS** Employee engagement is the level of commitment and involvement an employee has toward the company and its values. A national polling company claims that only 29% of U.S. employees feel engaged at work. You survey a random sample of 50 U.S. employees.

**a.** What can you conclude about the accuracy of the claim that the population proportion is 0.29 when 16 employees feel engaged at work?

**b.** What can you conclude about the accuracy of the claim that the population proportion is 0.29 when 23 employees feel engaged at work?

**c.** Assume that the true population proportion is 0.29. Estimate the variation among sample proportions for samples of size 50.

**In Exercises 11–16, find the margin of error for a survey that has the given sample size. Round your answer to the nearest tenth of a percent.**

**11.** 260

**12.** 1000

**13.** 2024

**14.** 6400

**15.** 3275

**16.** 750

**17. ATTENDING TO PRECISION** In a survey of 1020 U.S. adults, 41% said that their top priority for saving is retirement. *(See Example 4.)*

**a.** What is the margin of error for the survey?

**b.** Give an interval that is likely to contain the exact percent of all U.S. adults whose top priority for saving is retirement.

**18. ATTENDING TO PRECISION** In a survey of 1022 U.S. adults, 76% said that more emphasis should be placed on producing domestic energy from solar power.

**a.** What is the margin of error for the survey?

**b.** Give an interval that is likely to contain the exact percent of all U.S. adults who think more emphasis should be placed on producing domestic energy from solar power.

**19. ERROR ANALYSIS** In a survey, 8% of adult Internet users said they participate in sports fantasy leagues online. The margin of error is ±4%. Describe and correct the error in calculating the sample size.

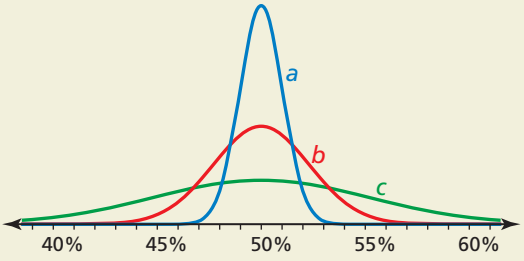
✗
$$\pm 0.08 = \pm\frac{1}{\sqrt{n}}$$
$$0.0064 = \frac{1}{n}$$
$$n \approx 156$$

**20. ERROR ANALYSIS** In a random sample of 2500 consumers, 61% prefer Game A over Game B. Describe and correct the error in giving an interval that is likely to contain the exact percent of all consumers who prefer Game A over Game B.

✗
$$\text{Margin of error} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{2500}} = 0.02$$
It is likely that the exact percent of all consumers who prefer Game A over Game B is between 60% and 62%.

**21.** **MAKING AN ARGUMENT** Your friend states that it is possible to have a margin of error between 0 and 100 percent, not including 0 or 100 percent. Is your friend correct? Explain your reasoning.

**22.** **HOW DO YOU SEE IT?** The figure shows the distribution of the sample proportions from three simulations using different sample sizes. Which simulation has the least margin of error? the greatest? Explain your reasoning.




**23.** **REASONING** A developer claims that the percent of city residents who favor building a new football stadium is likely between 52.3% and 61.7%. How many residents were surveyed?

**24.** **ABSTRACT REASONING** Suppose a random sample of size $n$ is required to produce a margin of error of $\pm E$. Write an expression in terms of $n$ for the sample size needed to reduce the margin of error to $\pm\frac{1}{2}E$. How many times must the sample size be increased to cut the margin of error in half? Explain.

**25.** **PROBLEM SOLVING** A survey reported that 47% of the voters surveyed, or about 235 voters, said they voted for Candidate A and the remainder said they voted for Candidate B.

**a.** How many voters were surveyed?

**b.** What is the margin of error for the survey?

**c.** For each candidate, find an interval that is likely to contain the exact percent of all voters who voted for the candidate.

**d.** Based on your intervals in part (c), can you be confident that Candidate B won? If not, how many people in the sample would need to vote for Candidate B for you to be confident that Candidate B won? (*Hint:* Find the least number of voters for Candidate B so that the intervals do not overlap.)

**26.** **THOUGHT PROVOKING** Consider a large population in which $\rho$ percent (in decimal form) have a certain characteristic. To be reasonably sure that you are choosing a sample that is representative of a population, you should choose a random sample of $n$ people where

$$n > 9\left(\frac{1-\rho}{\rho}\right).$$

**a.** Suppose $\rho = 0.5$. How large does $n$ need to be?

**b.** Suppose $\rho = 0.01$. How large does $n$ need to be?

**c.** What can you conclude from parts (a) and (b)?

**27.** **CRITICAL THINKING** In a survey, 52% of the respondents said they prefer sports drink X and 48% said they prefer sports drink Y. How many people would have to be surveyed for you to be confident that sports drink X is truly preferred by more than half the population? Explain.

## Maintaining Mathematical Proficiency

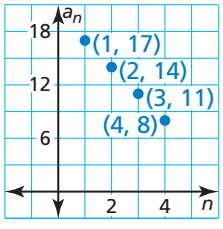
Reviewing what you learned in previous grades and lessons

**Find the inverse of the function.** *(Section 6.3)*

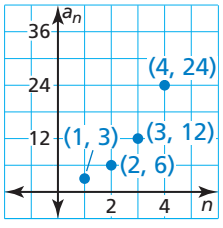
**28.** $y = 10^{x-3}$ **29.** $y = 2^x - 5$ **30.** $y = \ln(x + 5)$ **31.** $y = \log_6 x - 1$

**Determine whether the graph represents an arithmetic sequence or a geometric sequence. Then write a rule for the *n*th term.** *(Section 8.2 and Section 8.3)*

**32.**




**33.**




**34.**