

1.6 Describing Pairs of Angles

Essential Question How can you describe angle pair relationships and use these descriptions to find angle measures?

EXPLORATION 1 Finding Angle Measures

Work with a partner. The five-pointed star has a regular pentagon at its center.

- a. What do you notice about the following angle pairs?

x° and y°

y° and z°

x° and z°

- b. Find the values of the indicated variables. Do not use a protractor to measure the angles.

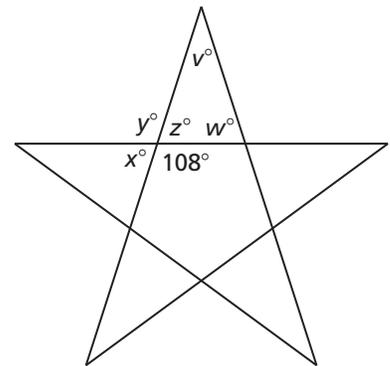
$x =$

$y =$

$z =$

$w =$

$v =$



Explain how you obtained each answer.

EXPLORATION 2 Finding Angle Measures

Work with a partner. A square is divided by its diagonals into four triangles.

- a. What do you notice about the following angle pairs?

a° and b°

c° and d°

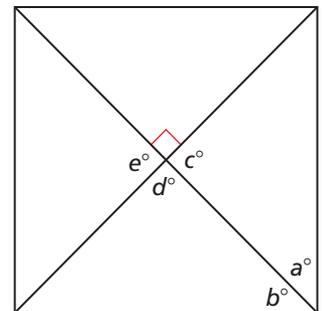
c° and e°

- b. Find the values of the indicated variables. Do not use a protractor to measure the angles.

$c =$

$d =$

$e =$



Explain how you obtained each answer.

ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

Communicate Your Answer

- How can you describe angle pair relationships and use these descriptions to find angle measures?
- What do you notice about the angle measures of complementary angles, supplementary angles, and vertical angles?

1.6 Lesson

Core Vocabulary

complementary angles, p. 48
 supplementary angles, p. 48
 adjacent angles, p. 48
 linear pair, p. 50
 vertical angles, p. 50

Previous

vertex
 sides of an angle
 interior of an angle
 opposite rays

What You Will Learn

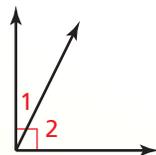
- ▶ Identify complementary and supplementary angles.
- ▶ Identify linear pairs and vertical angles.

Using Complementary and Supplementary Angles

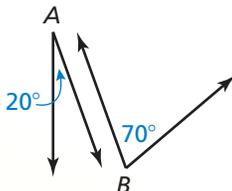
Pairs of angles can have special relationships. The measurements of the angles or the positions of the angles in the pair determine the relationship.

Core Concept

Complementary and Supplementary Angles



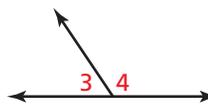
$\angle 1$ and $\angle 2$



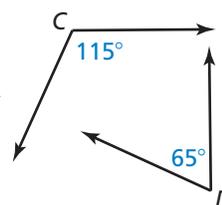
$\angle A$ and $\angle B$

complementary angles

Two positive angles whose measures have a sum of 90° . Each angle is the *complement* of the other.



$\angle 3$ and $\angle 4$



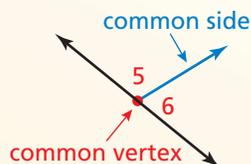
$\angle C$ and $\angle D$

supplementary angles

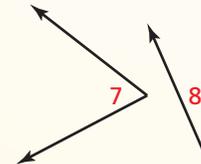
Two positive angles whose measures have a sum of 180° . Each angle is the *supplement* of the other.

Adjacent Angles

Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.



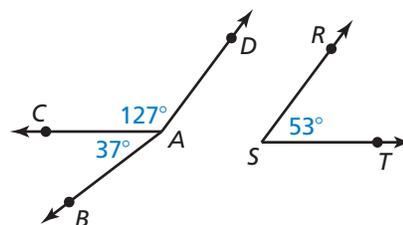
$\angle 5$ and $\angle 6$ are adjacent angles.



$\angle 7$ and $\angle 8$ are nonadjacent angles.

EXAMPLE 1 Identifying Pairs of Angles

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



SOLUTION

Because $37^\circ + 53^\circ = 90^\circ$, $\angle BAC$ and $\angle RST$ are complementary angles.

Because $127^\circ + 53^\circ = 180^\circ$, $\angle CAD$ and $\angle RST$ are supplementary angles.

Because $\angle BAC$ and $\angle CAD$ share a common vertex and side, they are adjacent angles.

COMMON ERROR

In Example 1, $\angle DAC$ and $\angle DAB$ share a common vertex and a common side. But they also share common interior points. So, they are *not* adjacent angles.

COMMON ERROR

Do not confuse angle names with angle measures.

EXAMPLE 2 Finding Angle Measures

- $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 62^\circ$. Find $m\angle 2$.
- $\angle 3$ is a supplement of $\angle 4$, and $m\angle 4 = 47^\circ$. Find $m\angle 3$.

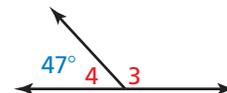
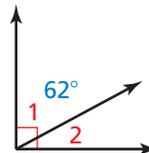
SOLUTION

- Draw a diagram with complementary adjacent angles to illustrate the relationship.

$$m\angle 2 = 90^\circ - m\angle 1 = 90^\circ - 62^\circ = 28^\circ$$

- Draw a diagram with supplementary adjacent angles to illustrate the relationship.

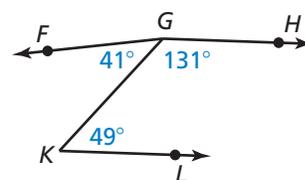
$$m\angle 3 = 180^\circ - m\angle 4 = 180^\circ - 47^\circ = 133^\circ$$



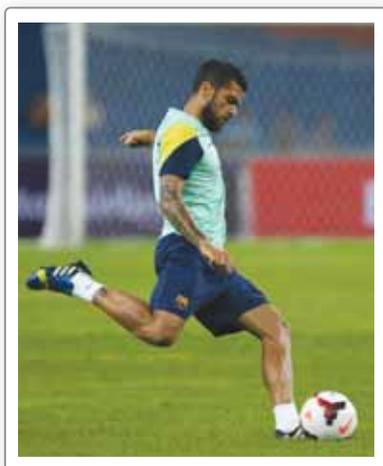
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In Exercises 1 and 2, use the figure.

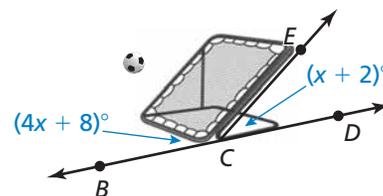
- Name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.
- Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.
- $\angle 1$ is a complement of $\angle 2$, and $m\angle 2 = 5^\circ$. Find $m\angle 1$.
- $\angle 3$ is a supplement of $\angle 4$, and $m\angle 3 = 148^\circ$. Find $m\angle 4$.



EXAMPLE 3 Real-Life Application



When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



SOLUTION

Step 1 Use the fact that the sum of the measures of supplementary angles is 180° .

$$m\angle BCE + m\angle ECD = 180^\circ$$

Write an equation.

$$(4x + 8)^\circ + (x + 2)^\circ = 180^\circ$$

Substitute angle measures.

$$5x + 10 = 180$$

Combine like terms.

$$x = 34$$

Solve for x .

Step 2 Evaluate the given expressions when $x = 34$.

$$m\angle BCE = (4x + 8)^\circ = (4 \cdot 34 + 8)^\circ = 144^\circ$$

$$m\angle ECD = (x + 2)^\circ = (34 + 2)^\circ = 36^\circ$$

► So, $m\angle BCE = 144^\circ$ and $m\angle ECD = 36^\circ$.

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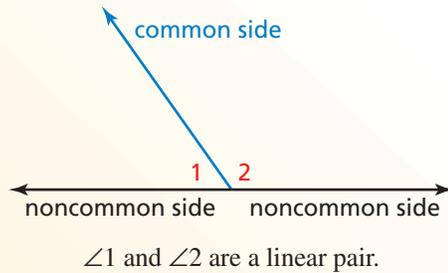
- $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles when $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$.

Using Other Angle Pairs

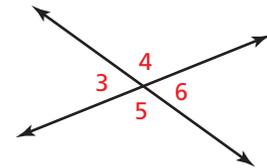
Core Concept

Linear Pairs and Vertical Angles

Two adjacent angles are a **linear pair** when their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.



Two angles are **vertical angles** when their sides form two pairs of opposite rays.



$\angle 3$ and $\angle 6$ are vertical angles.
 $\angle 4$ and $\angle 5$ are vertical angles.

COMMON ERROR

In Example 4, one side of $\angle 1$ and one side of $\angle 3$ are opposite rays. But the angles are not a linear pair because they are *nonadjacent*.

EXAMPLE 4 Identifying Angle Pairs

Identify all the linear pairs and all the vertical angles in the figure.

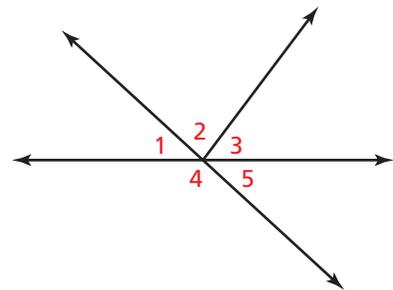
SOLUTION

To find vertical angles, look for angles formed by intersecting lines.

▶ $\angle 1$ and $\angle 5$ are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

▶ $\angle 1$ and $\angle 4$ are a linear pair. $\angle 4$ and $\angle 5$ are also a linear pair.

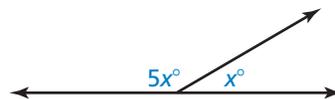


EXAMPLE 5 Finding Angle Measures in a Linear Pair

Two angles form a linear pair. The measure of one angle is five times the measure of the other angle. Find the measure of each angle.

SOLUTION

Step 1 Draw a diagram. Let x° be the measure of one angle. The measure of the other angle is $5x^\circ$.



Step 2 Use the fact that the angles of a linear pair are supplementary to write an equation.

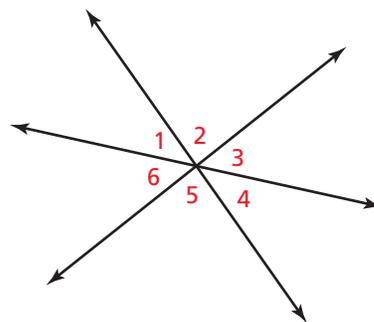
$$x^\circ + 5x^\circ = 180^\circ \quad \text{Write an equation.}$$

$$6x = 180 \quad \text{Combine like terms.}$$

$$x = 30 \quad \text{Divide each side by 6.}$$

▶ The measures of the angles are 30° and $5(30^\circ) = 150^\circ$.

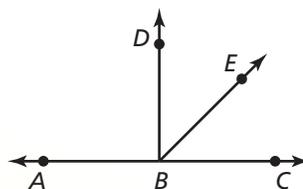
- Do any of the numbered angles in the figure form a linear pair? Which angles are vertical angles? Explain your reasoning.
- The measure of an angle is twice the measure of its complement. Find the measure of each angle.
- Two angles form a linear pair. The measure of one angle is $1\frac{1}{2}$ times the measure of the other angle. Find the measure of each angle.



Concept Summary

Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you **can conclude** from the diagram below.



YOU CAN CONCLUDE

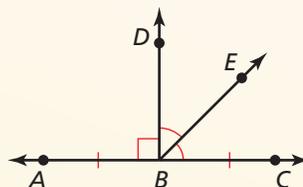
- All points shown are coplanar.
- Points A , B , and C are collinear, and B is between A and C .
- \overleftrightarrow{AC} , \overleftrightarrow{BD} , and \overleftrightarrow{BE} intersect at point B .
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point E lies in the interior of $\angle DBC$.

Here are some things you **cannot conclude** from the diagram above.

YOU CANNOT CONCLUDE

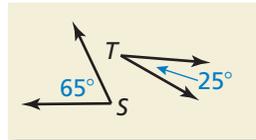
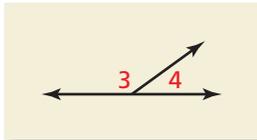
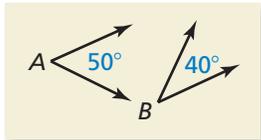
- $\overline{AB} \cong \overline{BC}$.
- $\angle DBE \cong \angle EBC$.
- $\angle ABD$ is a right angle.

To make such conclusions, the following information must be given.



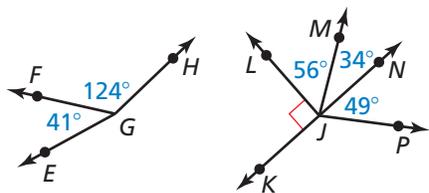
Vocabulary and Core Concept Check

- WRITING** Explain what is different between adjacent angles and vertical angles.
- WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.



Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use the figure. (See Example 1.)

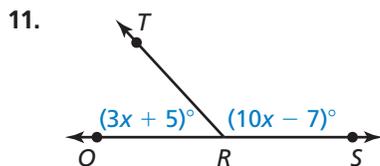


- Name a pair of adjacent complementary angles.
- Name a pair of adjacent supplementary angles.
- Name a pair of nonadjacent complementary angles.
- Name a pair of nonadjacent supplementary angles.

In Exercises 7–10, find the angle measure. (See Example 2.)

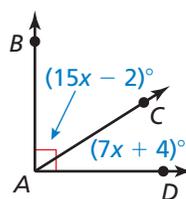
- $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 23^\circ$. Find $m\angle 2$.
- $\angle 3$ is a complement of $\angle 4$, and $m\angle 3 = 46^\circ$. Find $m\angle 4$.
- $\angle 5$ is a supplement of $\angle 6$, and $m\angle 5 = 78^\circ$. Find $m\angle 6$.
- $\angle 7$ is a supplement of $\angle 8$, and $m\angle 7 = 109^\circ$. Find $m\angle 8$.

In Exercises 11–14, find the measure of each angle. (See Example 3.)



11.

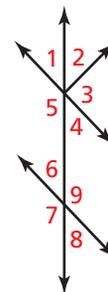
12.



- $\angle UVW$ and $\angle XYZ$ are complementary angles, $m\angle UVW = (x - 10)^\circ$, and $m\angle XYZ = (4x - 10)^\circ$.
- $\angle EFG$ and $\angle LMN$ are supplementary angles, $m\angle EFG = (3x + 17)^\circ$, and $m\angle LMN = (\frac{1}{2}x - 5)^\circ$.

In Exercises 15–18, use the figure. (See Example 4.)

- Identify the linear pair(s) that include $\angle 1$.
- Identify the linear pair(s) that include $\angle 7$.
- Are $\angle 6$ and $\angle 8$ vertical angles? Explain your reasoning.
- Are $\angle 2$ and $\angle 5$ vertical angles? Explain your reasoning.

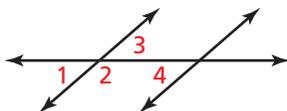


In Exercises 19–22, find the measure of each angle. (See Example 5.)

- Two angles form a linear pair. The measure of one angle is twice the measure of the other angle.
- Two angles form a linear pair. The measure of one angle is $\frac{1}{3}$ the measure of the other angle.
- The measure of an angle is nine times the measure of its complement.

22. The measure of an angle is $\frac{1}{4}$ the measure of its complement.

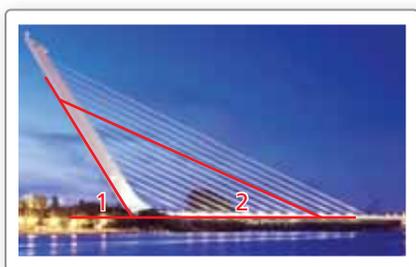
ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in identifying pairs of angles in the figure.



23.  $\angle 2$ and $\angle 4$ are adjacent.

24.  $\angle 1$ and $\angle 3$ form a linear pair.

In Exercises 25 and 26, the picture shows the Alamillo Bridge in Seville, Spain. In the picture, $m\angle 1 = 58^\circ$ and $m\angle 2 = 24^\circ$.



25. Find the measure of the supplement of $\angle 1$.
26. Find the measure of the supplement of $\angle 2$.
27. **PROBLEM SOLVING** The arm of a crossing gate moves 42° from a vertical position. How many more degrees does the arm have to move so that it is horizontal?



- A. 42° B. 138°
C. 48° D. 90°

28. **REASONING** The four lines of a baseball field intersect at home plate to form a right angle. A batter hits a fair ball such that the path of the baseball forms an angle of 27° with the third base foul line. What is the measure of the angle between the first base foul line and the path of the baseball?

29. **CONSTRUCTION** Construct a linear pair where one angle measure is 115° .
30. **CONSTRUCTION** Construct a pair of adjacent angles that have angle measures of 45° and 97° .

31. **PROBLEM SOLVING** $m\angle U = 2x^\circ$, and $m\angle V = 4m\angle U$. Which value of x makes $\angle U$ and $\angle V$ complements of each other?
A. 25 B. 9 C. 36 D. 18

MATHEMATICAL CONNECTIONS In Exercises 32–35, write and solve an algebraic equation to find the measure of each angle based on the given description.

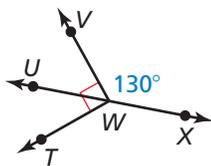
32. The measure of an angle is 6° less than the measure of its complement.
33. The measure of an angle is 12° more than twice the measure of its complement.
34. The measure of one angle is 3° more than $\frac{1}{2}$ the measure of its supplement.
35. Two angles form a linear pair. The measure of one angle is 15° less than $\frac{2}{3}$ the measure of the other angle.

CRITICAL THINKING In Exercises 36–41, tell whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

36. Complementary angles are adjacent.
37. Angles in a linear pair are supplements of each other.
38. Vertical angles are adjacent.
39. Vertical angles are supplements of each other.
40. If an angle is acute, then its complement is greater than its supplement.
41. If two complementary angles are congruent, then the measure of each angle is 45° .
42. **WRITING** Explain why the supplement of an acute angle must be obtuse.
43. **WRITING** Explain why an obtuse angle does not have a complement.

44. **THOUGHT PROVOKING** Sketch an intersection of roads. Identify any supplementary, complementary, or vertical angles.

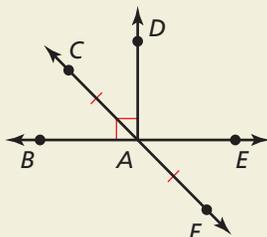
45. **ATTENDING TO PRECISION** Use the figure.



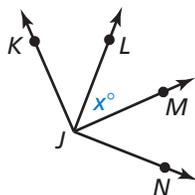
- Find $m\angle UWV$, $m\angle TWU$, and $m\angle TWX$.
- You write the measures of $\angle TWU$, $\angle TWX$, $\angle UWV$, and $\angle VWX$ on separate pieces of paper and place the pieces of paper in a box. Then you pick two pieces of paper out of the box at random. What is the probability that the angle measures you choose are supplementary? Explain your reasoning.

46. **HOW DO YOU SEE IT?** Tell whether you can conclude that each statement is true based on the figure. Explain your reasoning.

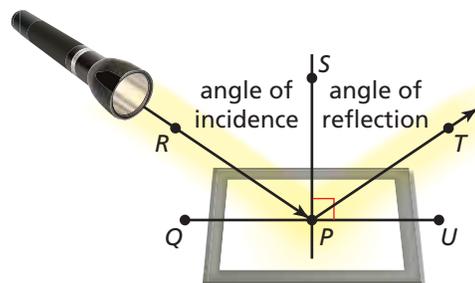
- $\overline{CA} \cong \overline{AF}$.
- Points C, A, and F are collinear.
- $\angle CAD \cong \angle EAF$.
- $\overline{BA} \cong \overline{AE}$.
- \overrightarrow{CF} , \overrightarrow{BE} , and \overrightarrow{AD} intersect at point A.
- $\angle BAC$ and $\angle CAD$ are complementary angles.
- $\angle DAE$ is a right angle.



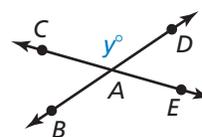
47. **REASONING** $\angle KJL$ and $\angle LJM$ are complements, and $\angle MJN$ and $\angle LJM$ are complements. Can you show that $\angle KJL \cong \angle MJN$? Explain your reasoning.



48. **MAKING AN ARGUMENT** Light from a flashlight strikes a mirror and is reflected so that the angle of reflection is congruent to the angle of incidence. Your classmate claims that $\angle QPR$ is congruent to $\angle TPU$ regardless of the measure of $\angle RPS$. Is your classmate correct? Explain your reasoning.



49. **DRAWING CONCLUSIONS** Use the figure.



- Write expressions for the measures of $\angle BAE$, $\angle DAE$, and $\angle CAB$.
 - What do you notice about the measures of vertical angles? Explain your reasoning.
50. **MATHEMATICAL CONNECTIONS** Let $m\angle 1 = x^\circ$, $m\angle 2 = y_1^\circ$, and $m\angle 3 = y_2^\circ$. $\angle 2$ is the complement of $\angle 1$, and $\angle 3$ is the supplement of $\angle 1$.
- Write equations for y_1 as a function of x and for y_2 as a function of x . What is the domain of each function? Explain.
 - Graph each function and describe its range.
51. **MATHEMATICAL CONNECTIONS** The sum of the measures of two complementary angles is 74° greater than the difference of their measures. Find the measure of each angle. Explain how you found the angle measures.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

(Skills Review Handbook)

- | | |
|--|---|
| 52. An integer is a whole number. | 53. An integer is an irrational number. |
| 54. An irrational number is a real number. | 55. A whole number is negative. |
| 56. A rational number is an integer. | 57. A natural number is an integer. |
| 58. A whole number is a rational number. | 59. An irrational number is negative. |