2.1 Conditional Statements

Essential Question

When is a conditional statement true or false?

A *conditional statement*, symbolized by $p \rightarrow q$, can be written as an "if-then statement" in which p is the *hypothesis* and q is the *conclusion*. Here is an example.

$$\underbrace{\textit{If a polygon is a triangle,}}_{\text{hypothesis, } p} \ \underbrace{\textit{then the sum of its angle measures is } 180°}_{\text{conclusion, } q}.$$

EXPLORATION 1 Determining Whether a Statement Is True or False

Work with a partner. A hypothesis can either be true or false. The same is true of a conclusion. For a conditional statement to be true, the hypothesis and conclusion do not necessarily both have to be true. Determine whether each conditional statement is true or false. Justify your answer.

a. If yesterday was Wednesday, then today is Thursday.

b. If an angle is acute, then it has a measure of 30°.

c. If a month has 30 days, then it is June.

d. If an even number is not divisible by 2, then 9 is a perfect cube.

EXPLORATION 2 Determining Whether a Statement Is True or False

Work with a partner. Use the points in the coordinate plane to determine whether each statement is true or false. Justify your answer.

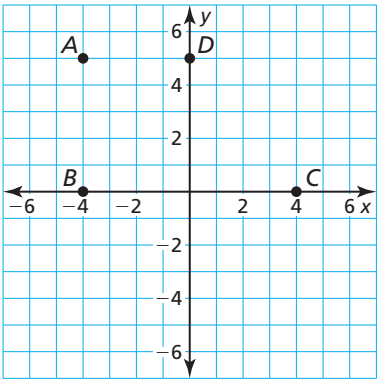
a. $\triangle ABC$ is a right triangle.

b. $\triangle BDC$ is an equilateral triangle.

c. $\triangle BDC$ is an isosceles triangle.

d. Quadrilateral $ABCD$ is a trapezoid.

e. Quadrilateral $ABCD$ is a parallelogram.



CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to distinguish correct logic or reasoning from that which is flawed.

EXPLORATION 3 Determining Whether a Statement Is True or False

Work with a partner. Determine whether each conditional statement is true or false. Justify your answer.

a. If $\triangle ADC$ is a right triangle, then the Pythagorean Theorem is valid for $\triangle ADC$.

b. If $\angle A$ and $\angle B$ are complementary, then the sum of their measures is 180°.

c. If figure $ABCD$ is a quadrilateral, then the sum of its angle measures is 180°.

d. If points A, B, and C are collinear, then they lie on the same line.

e. If $\overleftrightarrow{AB}$ and $\overleftrightarrow{BD}$ intersect at a point, then they form two pairs of vertical angles.

Communicate Your Answer

4. When is a conditional statement true or false?

5. Write one true conditional statement and one false conditional statement that are different from those given in Exploration 3. Justify your answer.

2.1 Lesson

Core Vocabulary

conditional statement, *p. 66*
if-then form, *p. 66*
hypothesis, *p. 66*
conclusion, *p. 66*
negation, *p. 66*
converse, *p. 67*
inverse, *p. 67*
contrapositive, *p. 67*
equivalent statements, *p. 67*
perpendicular lines, *p. 68*
biconditional statement, *p. 69*
truth value, *p. 70*
truth table, *p. 70*

What You Will Learn

- Write conditional statements.
- Use definitions written as conditional statements.
- Write biconditional statements.
- Make truth tables.

Writing Conditional Statements

Core Concept

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis* p and a *conclusion* q. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

Words If p, then q. **Symbols** $p \rightarrow q$ (read as "p implies q")

EXAMPLE 1 Rewriting a Statement in If-Then Form

Use red to identify the hypothesis and blue to identify the conclusion. Then rewrite the conditional statement in if-then form.

a. All birds have feathers.

b. You are in Texas if you are in Houston.

SOLUTION

a. All birds have feathers.

▶ If an animal is a bird, then it has feathers.

b. You are in Texas if you are in Houston.

▶ If you are in Houston, then you are in Texas.

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Use red to identify the hypothesis and blue to identify the conclusion. Then rewrite the conditional statement in if-then form.

1. All 30° angles are acute angles.

2. $2x + 7 = 1$, because $x = -3$.

Core Concept

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement p, you write the symbol for negation ($\sim$) before the letter. So, "not p" is written $\sim p$.

Words not p **Symbols** $\sim p$

EXAMPLE 2 Writing a Negation

Write the negation of each statement.

a. The ball is red.

b. The cat is *not* black.

SOLUTION

a. The ball is *not* red.

b. The cat is black.

Core Concept

Related Conditionals

Consider the conditional statement below.

Words If p, then q. **Symbols** $p \rightarrow q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q, then p. **Symbols** $q \rightarrow p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p, then not q. **Symbols** $\sim p \rightarrow \sim q$

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q, then not p. **Symbols** $\sim q \rightarrow \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

COMMON ERROR

Just because a conditional statement and its contrapositive are both true does not mean that its converse and inverse are both false. The converse and inverse could also both be true.

EXAMPLE 3 Writing Related Conditional Statements

Let p be "you are a guitar player" and let q be "you are a musician." Write each statement in words. Then decide whether it is *true* or *false*.

a. the conditional statement $p \rightarrow q$

b. the converse $q \rightarrow p$

c. the inverse $\sim p \rightarrow \sim q$

d. the contrapositive $\sim q \rightarrow \sim p$

SOLUTION

a. Conditional: If you are a guitar player, then you are a musician.
true; Guitar players are musicians.

b. Converse: If you are a musician, then you are a guitar player.
false; Not all musicians play the guitar.

c. Inverse: If you are not a guitar player, then you are not a musician.
false; Even if you do not play a guitar, you can still be a musician.

d. Contrapositive: If you are not a musician, then you are not a guitar player.
true; A person who is not a musician cannot be a guitar player.

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In Exercises 3 and 4, write the negation of the statement.

3. The shirt is green.

4. The shoes are *not* red.

5. Repeat Example 3. Let p be "the stars are visible" and let q be "it is night."

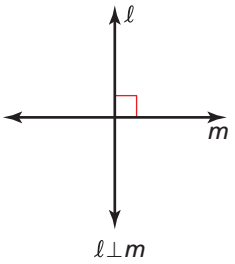
Using Definitions

You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of *perpendicular lines*.

If two lines intersect to form a right angle, then they are **perpendicular lines**.

You can also write the definition using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



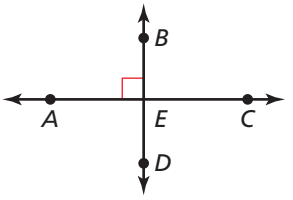
EXAMPLE 4 Using Definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.



SOLUTION

a. This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So, you can say the lines are perpendicular.

b. This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.

c. This statement is *false*. Point E does not lie on the same line as A and B, so the rays are not opposite rays.

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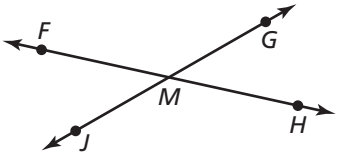
Use the diagram. Decide whether the statement is true. Explain your answer using the definitions you have learned.



6. $\angle JMF$ and $\angle FMG$ are supplementary.

7. Point M is the midpoint of $\overline{FH}$.

8. $\angle JMF$ and $\angle HMG$ are vertical angles.

9. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$

Writing Biconditional Statements

Core Concept

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

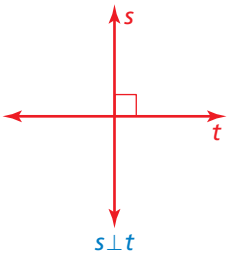
EXAMPLE 5 Writing a Biconditional Statement

Rewrite the definition of perpendicular lines as a single biconditional statement.

Definition If two lines intersect to form a right angle, then they are perpendicular lines.

SOLUTION

Let p be "two lines intersect to form a right angle" and let q be "they are perpendicular lines."
Use red to identify p and blue to identify q.
Write the definition $p \rightarrow q$.



Definition If two lines intersect to form a right angle, then they are perpendicular lines.

Write the converse $q \rightarrow p$.

Converse If two lines are perpendicular lines, then they intersect to form a right angle.

Use the definition and its converse to write the biconditional statement $p \leftrightarrow q$.

▶ **Biconditional** Two lines intersect to form a right angle if and only if they are perpendicular lines.

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10. Rewrite the definition of a right angle as a single biconditional statement.

Definition If an angle is a right angle, then its measure is 90°.

11. Rewrite the definition of congruent segments as a single biconditional statement.

Definition If two line segments have the same length, then they are congruent segments.

12. Rewrite the statements as a single biconditional statement.

If Mary is in theater class, then she will be in the fall play. If Mary is in the fall play, then she must be taking theater class.

13. Rewrite the statements as a single biconditional statement.

If you can run for President, then you are at least 35 years old. If you are at least 35 years old, then you can run for President.

Making Truth Tables

The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table below shows the truth values for hypothesis p and conclusion q.

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The conditional statement $p \rightarrow q$ is only false when a true hypothesis produces a false conclusion.

Two statements are *logically equivalent* when they have the same truth table.

EXAMPLE 6 Making a Truth Table

Use the truth table above to make truth tables for the converse, inverse, and contrapositive of a conditional statement $p \rightarrow q$.

SOLUTION

The truth tables for the converse and the inverse are shown below. Notice that the converse and the inverse are logically equivalent because they have the same truth table.

Converse		
p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

The truth table for the contrapositive is shown below. Notice that a conditional statement and its contrapositive are logically equivalent because they have the same truth table.

Contrapositive				
p	q	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

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14. Make a truth table for the conditional statement $p \rightarrow \sim q$.

15. Make a truth table for the conditional statement $\sim(p \rightarrow q)$.

2.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What type of statements are either both true or both false?

2. **WHICH ONE DOESN'T BELONG?** Which statement does *not* belong with the other three? Explain your reasoning.

If today is Tuesday, then tomorrow is Wednesday.	If it is Independence Day, then it is July.
If an angle is acute, then its measure is less than 90°.	If you are an athlete, then you play soccer.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, copy the conditional statement. Underline the hypothesis and circle the conclusion.

3. If a polygon is a pentagon, then it has five sides.

4. If two lines form vertical angles, then they intersect.

5. If you run, then you are fast.

6. If you like math, then you like science.

In Exercises 7–12, rewrite the conditional statement in if-then form. *(See Example 1.)*

7. $9x + 5 = 23$, because $x = 2$.

8. Today is Friday, and tomorrow is the weekend.

9. You are in a band, and you play the drums.

10. Two right angles are supplementary angles.

11. Only people who are registered are allowed to vote.

12. The measures of complementary angles sum to 90°.

In Exercises 13–16, write the negation of the statement. *(See Example 2.)*

13. The sky is blue.

14. The lake is cold.

15. The ball is *not* pink.

16. The dog is *not* a Lab.

In Exercises 17–24, write the conditional statement $p \rightarrow q$, the converse $q \rightarrow p$, the inverse $\sim p \rightarrow \sim q$, and the contrapositive $\sim q \rightarrow \sim p$ in words. Then decide whether each statement is true or false. *(See Example 3.)*

17. Let p be "two angles are supplementary" and let q be "the measures of the angles sum to 180°."

18. Let p be "you are in math class" and let q be "you are in Geometry."

19. Let p be "you do your math homework" and let q be "you will do well on the test."

20. Let p be "you are not an only child" and let q be "you have a sibling."

21. Let p be "it does not snow" and let q be "I will run outside."

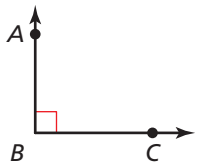
22. Let p be "the Sun is out" and let q be "it is daytime."

23. Let p be "$3x - 7 = 20$" and let q be "$x = 9$."

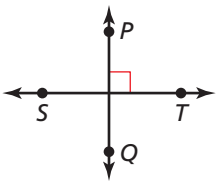
24. Let p be "it is Valentine's Day" and let q be "it is February."

In Exercises 25–28, decide whether the statement about the diagram is true. Explain your answer using the definitions you have learned. *(See Example 4.)*

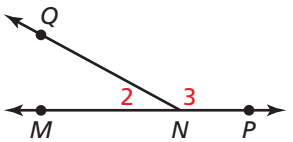
25. $m\angle ABC = 90°$



26. $\overleftrightarrow{PQ} \perp \overleftrightarrow{ST}$



27. $m\angle 2 + m\angle 3 = 180°$



28. M is the midpoint of $\overline{AB}$.

In Exercises 29–32, rewrite the definition of the term as a biconditional statement. *(See Example 5.)*

29. The *midpoint* of a segment is the point that divides the segment into two congruent segments.

30. Two angles are *vertical angles* when their sides form two pairs of opposite rays.

31. *Adjacent angles* are two angles that share a common vertex and side but have no common interior points.

32. Two angles are *supplementary angles* when the sum of their measures is 180°.

In Exercises 33–36, rewrite the statements as a single biconditional statement. *(See Example 5.)*

33. If a polygon has three sides, then it is a triangle.
If a polygon is a triangle, then it has three sides.

34. If a polygon has four sides, then it is a quadrilateral.
If a polygon is a quadrilateral, then it has four sides.

35. If an angle is a right angle, then it measures 90°.
If an angle measures 90°, then it is a right angle.

36. If an angle is obtuse, then it has a measure between 90° and 180°.
If an angle has a measure between 90° and 180°, then it is obtuse.

37. **ERROR ANALYSIS** Describe and correct the error in rewriting the conditional statement in if-then form.

Conditional statement
All high school students take four English courses.

If-then form
If a high school student takes four courses, then all four are English courses.

38. **ERROR ANALYSIS** Describe and correct the error in writing the converse of the conditional statement.

Conditional statement
If it is raining, then I will bring an umbrella.

Converse
If it is not raining, then I will not bring an umbrella.

In Exercises 39–44, create a truth table for the logical statement. *(See Example 6.)*

39. $\sim p \rightarrow q$

40. $\sim q \rightarrow p$

41. $\sim(\sim p \rightarrow \sim q)$

42. $\sim(p \rightarrow \sim q)$

43. $q \rightarrow \sim p$

44. $\sim(q \rightarrow p)$

45. **USING STRUCTURE** The statements below describe three ways that rocks are formed.

Igneous rock is formed from the cooling of molten rock.

Sedimentary rock is formed from pieces of other rocks.

Metamorphic rock is formed by changing temperature, pressure, or chemistry.

a. Write each statement in if-then form.

b. Write the converse of each of the statements in part (a). Is the converse of each statement true? Explain your reasoning.

c. Write a true if-then statement about rocks that is different from the ones in parts (a) and (b). Is the converse of your statement true or false? Explain your reasoning.

46. **MAKING AN ARGUMENT** Your friend claims the statement "If I bought a shirt, then I went to the mall" can be written as a true biconditional statement. Your sister says you cannot write it as a biconditional. Who is correct? Explain your reasoning.

47. **REASONING** You are told that the contrapositive of a statement is true. Will that help you determine whether the statement can be written as a true biconditional statement? Explain your reasoning.

48. **PROBLEM SOLVING** Use the conditional statement to identify the if-then statement as the converse, inverse, or contrapositive of the conditional statement. Then use the symbols to represent both statements.

Conditional statement
If I rode my bike to school, then I did not walk to school.

If-then statement
If I did not ride my bike to school, then I walked to school.

p	q	$\sim$	$\rightarrow$	$\leftrightarrow$

USING STRUCTURE **In Exercises 49–52, rewrite the conditional statement in if-then form. Then underline the hypothesis and circle the conclusion.**

49.

If you tell the truth, you don't have to remember anything.
Mark Twain

50.

You have to expect things of yourself before you can do them.
Michael Jordan

51.

If one is lucky, a solitary fantasy can totally transform one million realities.
Maya Angelou

52.

Whoever is happy will make others happy too.
Anne Frank

53. **MATHEMATICAL CONNECTIONS** Can the statement "If $x^2 - 10 = x + 2$, then $x = 4$" be combined with its converse to form a true biconditional statement?

54. **CRITICAL THINKING** The largest natural arch in the United States is Landscape Arch, located in Thompson, Utah. It spans 290 feet.

a. Use the information to write at least two true conditional statements.

b. Which type of related conditional statement must also be true? Write the related conditional statements.

c. What are the other two types of related conditional statements? Write the related conditional statements. Then determine their truth values. Explain your reasoning.

55. **REASONING** Which statement has the same meaning as the given statement?

Given statement
You can watch a movie after you do your homework.

(A) If you do your homework, then you can watch a movie afterward.

(B) If you do not do your homework, then you can watch a movie afterward.

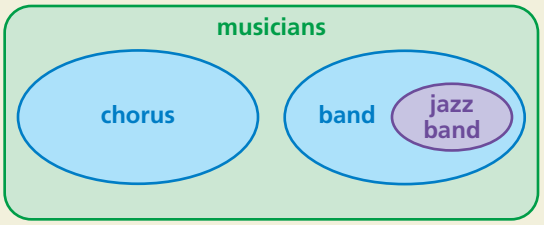
(C) If you cannot watch a movie afterward, then do your homework.

(D) If you can watch a movie afterward, then do not do your homework.

56. **THOUGHT PROVOKING** Write three conditional statements, where one is always true, one is always false, and one depends on the person interpreting the statement.

57. **CRITICAL THINKING** One example of a conditional statement involving dates is "If today is August 31, then tomorrow is September 1." Write a conditional statement using dates from two different months so that the truth value depends on when the statement is read.

58. **HOW DO YOU SEE IT?** The Venn diagram represents all the musicians at a high school. Write three conditional statements in if-then form describing the relationships between the various groups of musicians.



59. **MULTIPLE REPRESENTATIONS** Create a Venn diagram representing each conditional statement. Write the converse of each conditional statement. Then determine whether each conditional statement and its converse are true or false. Explain your reasoning.

a. If you go to the zoo to see a lion, then you will see a cat.

b. If you play a sport, then you wear a helmet.

c. If this month has 31 days, then it is not February.

60. **DRAWING CONCLUSIONS** You measure the heights of your classmates to get a data set.

a. Tell whether this statement is true: If x and y are the least and greatest values in your data set, then the mean of the data is between x and y.

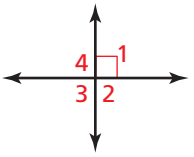
b. Write the converse of the statement in part (a). Is the converse true? Explain your reasoning.

c. Copy and complete the statement below using *mean*, *median*, or *mode* to make a conditional statement that is true for any data set. Explain your reasoning.

> If a data set has a mean, median, and a mode, then the _____ of the data set will always be a data value.

61. **WRITING** Write a conditional statement that is true, but its converse is false.

62. **CRITICAL THINKING** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90°$. Use the definition of linear pairs.



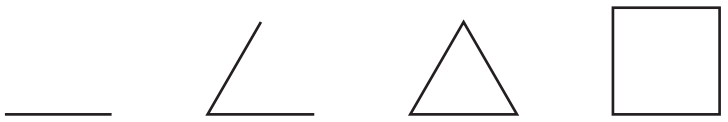
63. **WRITING** Advertising slogans such as "Buy these shoes! They will make you a better athlete!" often imply conditional statements. Find an advertisement or write your own slogan. Then write it as a conditional statement.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the pattern. Then draw the next two figures in the sequence. *(Skills Review Handbook)*

64.



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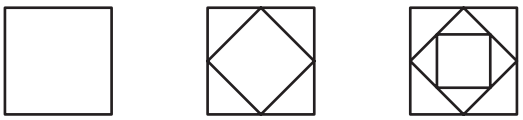

Find the pattern. Then write the next two numbers. *(Skills Review Handbook)*

66. 1, 3, 5, 7, . . .

67. 12, 23, 34, 45, . . .

68. $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \ldots$

69. 1, 4, 9, 16, . . .