# 11.2 Areas of Circles and Sectors

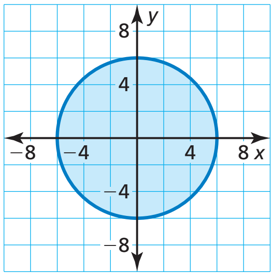
## Essential Question

How can you find the area of a sector of a circle?

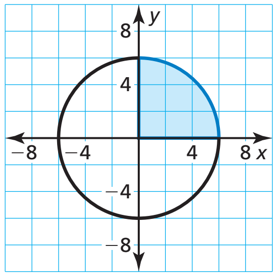
### EXPLORATION 1 Finding the Area of a Sector of a Circle

**Work with a partner.** A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. Find the area of each shaded circle or sector of a circle.

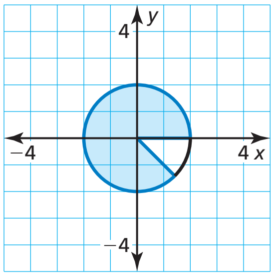
**a.** entire circle



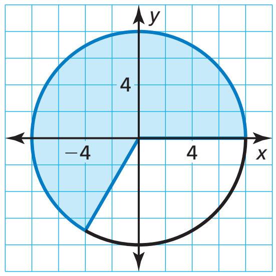

**b.** one-fourth of a circle




**c.** seven-eighths of a circle




**d.** two-thirds of a circle




**REASONING ABSTRACTLY**

To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

### EXPLORATION 2 Finding the Area of a Circular Sector

**Work with a partner.** A center pivot irrigation system consists of 400 meters of sprinkler equipment that rotates around a central pivot point at a rate of once every 3 days to irrigate a circular region with a diameter of 800 meters. Find the area of the sector that is irrigated by this system in one day.

## Communicate Your Answer

**3.** How can you find the area of a sector of a circle?

**4.** In Exploration 2, find the area of the sector that is irrigated in 2 hours.

# 11.2 Lesson

## Core Vocabulary

population density, *p. 603*
sector of a circle, *p. 604*

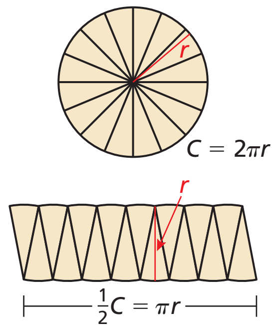
***Previous***
circle
radius
diameter
intercepted arc

## What You Will Learn

- Use the formula for the area of a circle.
- Use the formula for population density.
- Find areas of sectors.
- Use areas of sectors.

## Using the Formula for the Area of a Circle

You can divide a circle into congruent sections and rearrange the sections to form a figure that approximates a parallelogram. Increasing the number of congruent sections increases the figure's resemblance to a parallelogram.




The base of the parallelogram that the figure approaches is half of the circumference, so $b = \frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$. The height is the radius, so $h = r$. So, the area of the parallelogram is $A = bh = (\pi r)(r) = \pi r^2$.

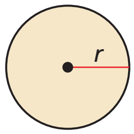
## Core Concept

### Area of a Circle

The area of a circle is

$$A = \pi r^2$$

where $r$ is the radius of the circle.




### EXAMPLE 1 Using the Formula for the Area of a Circle

Find each indicated measure.

**a.** area of a circle with a radius of 2.5 centimeters

**b.** diameter of a circle with an area of 113.1 square centimeters

**SOLUTION**

**a.**

| | |
|---|---|
| $A = \pi r^2$ | Formula for area of a circle |
| $= \pi \cdot (2.5)^2$ | Substitute 2.5 for $r$. |
| $= 6.25\pi$ | Simplify. |
| $\approx 19.63$ | Use a calculator. |

The area of the circle is about 19.63 square centimeters.

**b.**

| | |
|---|---|
| $A = \pi r^2$ | Formula for area of a circle |
| $113.1 = \pi r^2$ | Substitute 113.1 for $A$. |
| $\frac{113.1}{\pi} = r^2$ | Divide each side by $\pi$. |
| $6 \approx r$ | Find the positive square root of each side. |

The radius is about 6 centimeters, so the diameter is about 12 centimeters.

## Monitoring Progress

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1. Find the area of a circle with a radius of 4.5 meters.
2. Find the radius of a circle with an area of 176.7 square feet.

## Using the Formula for Population Density

The **population density** of a city, county, or state is a measure of how many people live within a given area.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}}$$

Population density is usually given in terms of square miles but can be expressed using other units, such as city blocks.

### EXAMPLE 2 Using the Formula for Population Density

**a.** About 430,000 people live in a 5-mile radius of a city's town hall. Find the population density in people per square mile.

**b.** A region with a 3-mile radius has a population density of about 6195 people per square mile. Find the number of people who live in the region.

**SOLUTION**

**a. Step 1** Find the area of the region.

$$A = \pi r^2 = \pi \cdot 5^2 = 25\pi$$

The area of the region is $25\pi \approx 78.54$ square miles.

**Step 2** Find the population density.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} \qquad \text{Formula for population density}$$

$$= \frac{430{,}000}{25\pi} \qquad \text{Substitute.}$$

$$\approx 5475 \qquad \text{Use a calculator.}$$

▶ The population density is about 5475 people per square mile.

**b. Step 1** Find the area of the region.

$$A = \pi r^2 = \pi \cdot 3^2 = 9\pi$$

The area of the region is $9\pi \approx 28.27$ square miles.

**Step 2** Let $x$ represent the number of people who live in the region. Find the value of $x$.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} \qquad \text{Formula for population density}$$

$$6195 \approx \frac{x}{9\pi} \qquad \text{Substitute.}$$

$$175{,}159 \approx x \qquad \text{Multiply and use a calculator.}$$

▶ The number of people who live in the region is about 175,159.

### Monitoring Progress

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**3.** About 58,000 people live in a region with a 2-mile radius. Find the population density in people per square mile.

**4.** A region with a 3-mile radius has a population density of about 1000 people per square mile. Find the number of people who live in the region.

## Finding Areas of Sectors

A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram below, sector $APB$ is bounded by $\overline{AP}$, $\overline{BP}$, and $\overset{\frown}{AB}$.

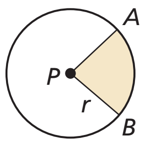
### ANALYZING RELATIONSHIPS

The area of a sector is a fractional part of the area of a circle. The area of a sector formed by a 45° arc is $\frac{45°}{360°}$, or $\frac{1}{8}$ of the area of the circle.

## Core Concept

### Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle ($\pi r^2$) is equal to the ratio of the measure of the intercepted arc to 360°.

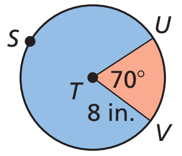



$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360°}, \text{ or}$$

$$\text{Area of sector } APB = \frac{m\overset{\frown}{AB}}{360°} \cdot \pi r^2$$

**EXAMPLE 3** **Finding Areas of Sectors**

Find the areas of the sectors formed by $\angle UTV$.




**SOLUTION**

**Step 1** Find the measures of the minor and major arcs.

Because $m\angle UTV = 70°$, $m\overset{\frown}{UV} = 70°$ and $m\overset{\frown}{USV} = 360° - 70° = 290°$.

**Step 2** Find the areas of the small and large sectors.

| | |
|---|---|
| $\text{Area of small sector} = \frac{m\overset{\frown}{UV}}{360°} \cdot \pi r^2$ | Formula for area of a sector |
| $= \frac{70°}{360°} \cdot \pi \cdot 8^2$ | Substitute. |
| $\approx 39.10$ | Use a calculator. |
| $\text{Area of large sector} = \frac{m\overset{\frown}{USV}}{360°} \cdot \pi r^2$ | Formula for area of a sector |
| $= \frac{290°}{360°} \cdot \pi \cdot 8^2$ | Substitute. |
| $\approx 161.97$ | Use a calculator. |

▶ The areas of the small and large sectors are about 39.10 square inches and about 161.97 square inches, respectively.

## Monitoring Progress

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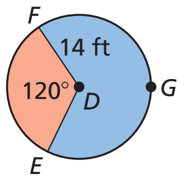
**Find the indicated measure.**

**5.** area of red sector

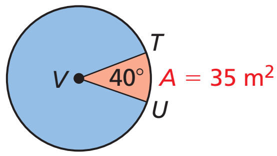
**6.** area of blue sector



## Using Areas of Sectors

### EXAMPLE 4 Using the Area of a Sector

Find the area of $\odot V$.




**SOLUTION**

$$\text{Area of sector } TVU = \frac{m\overparen{TU}}{360^\circ} \cdot \text{Area of } \odot V \quad \text{Formula for area of a sector}$$

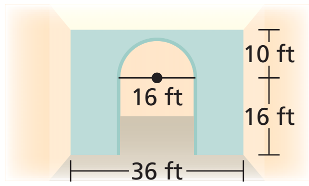
$$35 = \frac{40^\circ}{360^\circ} \cdot \text{Area of } \odot V \quad \text{Substitute.}$$

$$315 = \text{Area of } \odot V \quad \text{Solve for area of } \odot V.$$

▶ The area of $\odot V$ is 315 square meters.

### EXAMPLE 5 Finding the Area of a Region

A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?




**SOLUTION**

The area you need to paint is the area of the rectangle minus the area of the entrance. The entrance can be divided into a semicircle and a square.

$$\text{Area of wall} = \text{Area of rectangle} - (\text{Area of semicircle} + \text{Area of square})$$

$$= 36(26) - \left[\frac{180^\circ}{360^\circ} \cdot (\pi \cdot 8^2) + 16^2\right]$$

$$= 936 - (32\pi + 256)$$

$$\approx 579.47$$

▶ The area you need to paint is about 579 square feet.

**COMMON ERROR**

Use the radius (8 feet), not the diameter (16 feet), when you calculate the area of the semicircle.

### Monitoring Progress

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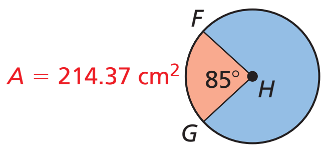
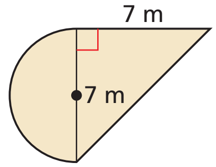
**7.** Find the area of $\odot H$.




**8.** Find the area of the figure.




**9.** If you know the area and radius of a sector of a circle, can you find the measure of the intercepted arc? Explain.

# 11.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

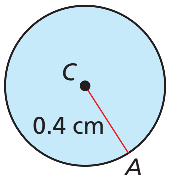
1. **VOCABULARY** A(n) __________ of a circle is the region bounded by two radii of the circle and their intercepted arc.

2. **WRITING** The arc measure of a sector in a given circle is doubled. Will the area of the sector also be doubled? Explain your reasoning.

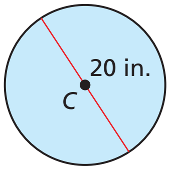
## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–10, find the indicated measure.** *(See Example 1.)*

3. area of $\odot C$




4. area of $\odot C$




5. area of a circle with a radius of 5 inches

6. area of a circle with a diameter of 16 feet

7. radius of a circle with an area of 89 square feet

8. radius of a circle with an area of 380 square inches

9. diameter of a circle with an area of 12.6 square inches

10. diameter of a circle with an area of $676\pi$ square centimeters

**In Exercises 11–14, find the indicated measure.** *(See Example 2.)*

11. About 210,000 people live in a region with a 12-mile radius. Find the population density in people per square mile.

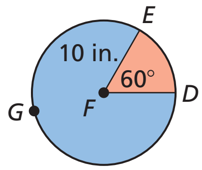
12. About 650,000 people live in a region with a 6-mile radius. Find the population density in people per square mile.

13. A region with a 4-mile radius has a population density of about 6366 people per square mile. Find the number of people who live in the region.

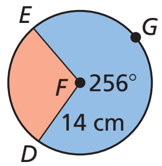
14. About 79,000 people live in a circular region with a population density of about 513 people per square mile. Find the radius of the region.

**In Exercises 15–18, find the areas of the sectors formed by $\angle DFE$.** *(See Example 3.)*

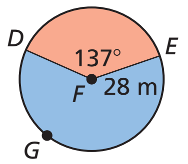
15.



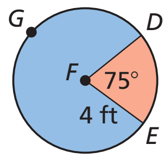

16.



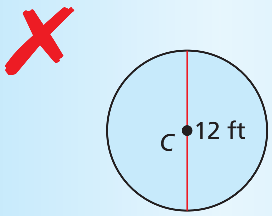

17.




18.




19. **ERROR ANALYSIS** Describe and correct the error in finding the area of the circle.

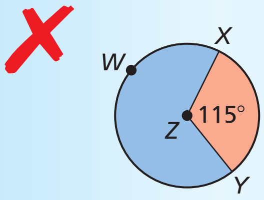



$A = \pi(12)^2$

$= 144\pi$

$\approx 452.39 \text{ ft}^2$

20. **ERROR ANALYSIS** Describe and correct the error in finding the area of sector $XZY$ when the area of $\odot Z$ is 255 square feet.



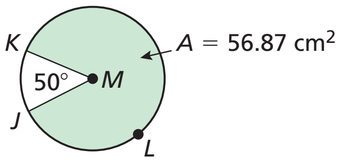

Let $n$ be the area of sector $XZY$.

$\frac{n}{360} = \frac{115}{255}$

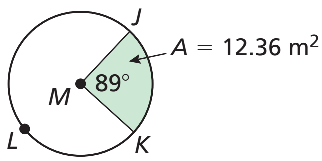
$n \approx 162.35 \text{ ft}^2$

**In Exercises 21 and 22, the area of the shaded sector is shown. Find the indicated measure.** *(See Example 4.)*

**21.** area of $\odot M$

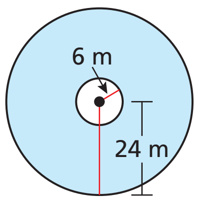



**22.** radius of $\odot M$

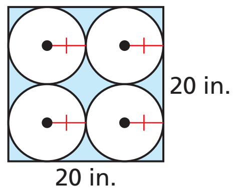



**In Exercises 23–28, find the area of the shaded region.** *(See Example 5.)*

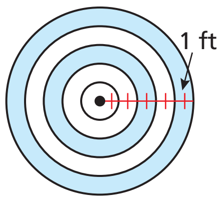
**23.**



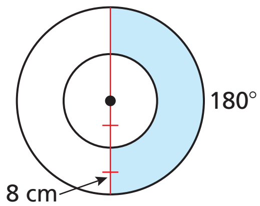

**24.**



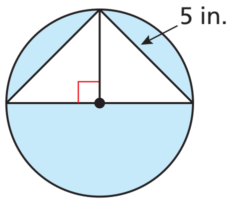

**25.**



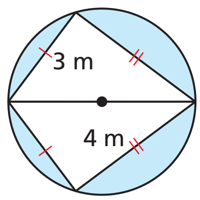

**26.**



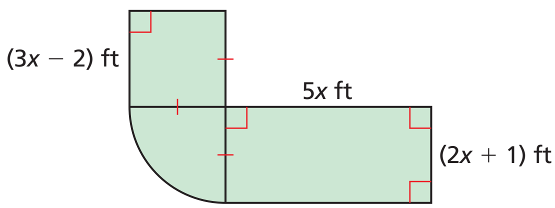

**27.**




**28.**

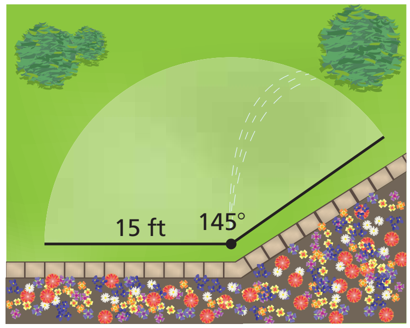



**29. PROBLEM SOLVING** The diagram shows the shape of a putting green at a miniature golf course. One part of the green is a sector of a circle. Find the area of the putting green.




**30. MAKING AN ARGUMENT** Your friend claims that if the radius of a circle is doubled, then its area doubles. Is your friend correct? Explain your reasoning.

**31. MODELING WITH MATHEMATICS** The diagram shows the area of a lawn covered by a water sprinkler.




**a.** What is the area of the lawn that is covered by the sprinkler?

**b.** The water pressure is weakened so that the radius is 12 feet. What is the area of the lawn that will be covered?

**32. MODELING WITH MATHEMATICS** The diagram shows a projected beam of light from a lighthouse.


**a.** What is the area of water that can be covered by the light from the lighthouse?

**b.** What is the area of land that can be covered by the light from the lighthouse?

**33. ANALYZING RELATIONSHIPS** Look back at the Perimeters of Similar Polygons Theorem (Theorem 8.1) and the Areas of Similar Polygons Theorem (Theorem 8.2) in Section 8.1. How would you rewrite these theorems to apply to circles? Explain your reasoning.

**34. ANALYZING RELATIONSHIPS** A square is inscribed in a circle. The same square is also circumscribed about a smaller circle. Draw a diagram that represents this situation. Then find the ratio of the area of the larger circle to the area of the smaller circle.

**35.** **CONSTRUCTION** The table shows how students get to school.

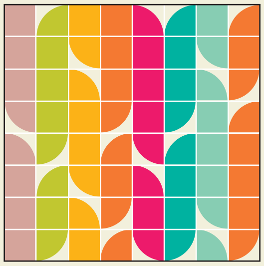
| Method | Percent of students |
|---|---|
| bus | 65% |
| walk | 25% |
| other | 10% |

**a.** Explain why a circle graph is appropriate for the data.

**b.** You will represent each method by a sector of a circle graph. Find the central angle to use for each sector. Then construct the graph using a radius of 2 inches.

**c.** Find the area of each sector in your graph.

**36.** **HOW DO YOU SEE IT?** The outermost edges of the pattern shown form a square. If you know the dimensions of the outer square, is it possible to compute the total colored area? Explain.



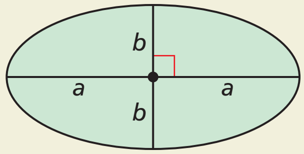
**37.** **ABSTRACT REASONING** A circular pizza with a 12-inch diameter is enough for you and 2 friends. You want to buy pizzas for yourself and 7 friends. A 10-inch diameter pizza with one topping costs $6.99 and a 14-inch diameter pizza with one topping costs $12.99. How many 10-inch and 14-inch pizzas should you buy in each situation? Explain.

**a.** You want to spend as little money as possible.

**b.** You want to have three pizzas, each with a different topping, and spend as little money as possible.

**c.** You want to have as much of the thick outer crust as possible.

**38.** **THOUGHT PROVOKING** You know that the area of a circle is $\pi r^2$. Find the formula for the area of an *ellipse*, shown below.




**39.** **MULTIPLE REPRESENTATIONS** Consider a circle with a radius of 3 inches.

**a.** Complete the table, where $x$ is the measure of the arc and $y$ is the area of the corresponding sector. Round your answers to the nearest tenth.

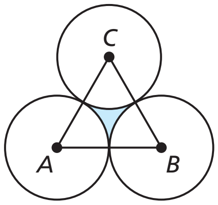
| $x$ | 30° | 60° | 90° | 120° | 150° | 180° |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**b.** Graph the data in the table.

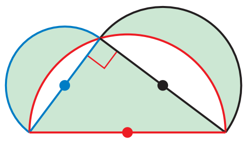
**c.** Is the relationship between $x$ and $y$ linear? Explain.

**d.** If parts (a)–(c) were repeated using a circle with a radius of 5 inches, would the areas in the table change? Would your answer to part (c) change? Explain your reasoning.

**40.** **CRITICAL THINKING** Find the area between the three congruent tangent circles. The radius of each circle is 6 inches.




**41.** **PROOF** Semicircles with diameters equal to three sides of a right triangle are drawn, as shown. Prove that the sum of the areas of the two shaded crescents equals the area of the triangle.

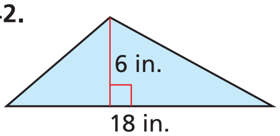


## Maintaining Mathematical Proficiency

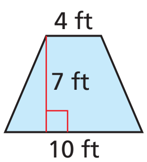
Reviewing what you learned in previous grades and lessons

**Find the area of the figure.** *(Skills Review Handbook)*

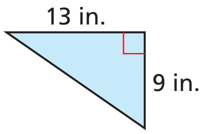
**42.**




**43.**




**44.**




**45.**